Econ 375
Problem Set 4

1. We did this one in class.

2. An increase in the saving rate will raise the steady-state level of income but it will have no effect on the steady-state rate of growth.

3. Consider an economy described by the production function: $Y = K^{.3}L^{.7}$
a) $y = k^{.3}$
b) $k^* = (s/d)^{1/.7}$; $y^* = (s/d)^{.3/.7}$; $c^* = (1-s)(s/d)^{.3/.7}$
c) A saving rate of s = 0 maximizes y*. A saving rate of s = .3 maximizes c*.

4. We did this one in class.

5. First, consider the steady-states. In the steady-state diagram, slower population growth rates shifts the line representing population growth and depreciation downward. The new steady state has a higher level of capital per worker, k*, and hence a higher level of output per worker. What about steady-state growth rates? In steady state, total output grows at rate n + g, whereas output per person grows at rate g. Hence, slower population growth will lower total output growth, but per person output growth will be the same.

Now consider the transition. We know that the steady-state level of output per person is higher with low population growth. Hence, during the transition to the new steady state, output per person must grow at a faster rate than g for a while. In the decades after the fall in population growth, growth in total output will fall while growth in output per person will rise.

6. Look on pages 236-238 for some answers.

7. Think about the political consequences of the implied transition process for each initial state.

8. See page 239-242 for the answer.

9. Country 1 has a more highly educated work force than country 2: $E_1 > E_2$.
a) In the Solow model, the rate of growth of total income is equal to n+g, which is independent of the work force's level of education. The two countries will, thus, have the same rate of growth of total income because they have the same population growth and technological progress.

b) Because both countries have the same saving rate, the same n and the same g, we know that the two countries will converge to the same steady-state level of capital per efficiency unit of labor k*. Hence, output per efficiency unit of labor in the steady-state will be the same in both countries. But $y^* = Y/(LE)$ or $Y/L = y^*E$. We know that y* will be the same in both countries, but that $E_1 > E_2$. Therefore, $y^*E_1 > y^*E_2$. This implies that $(Y/L_1) > (Y/L_2)$. Thus, the level of income per worker will be higher in the country with the more educated labor force.

c) We know that the real rental price of capital R equals the marginal product of capital (MPK). But the MPK depends on the capital stock per efficiency unit of labor. In the steady state, both countries have $k_1^* = k_2^* = k^*$ because both countries have the same s, n, and g. Therefore, it must be true that $R_1 = R_2 =$ MPK. Thus the real rental price of capital is identical in both countries.

d) Output is divided between capital income and labor income. Therefore the wage per efficiency unit of labor can be expressed as: w = f(k) - MPK x k. As discussed in parts (b) and (c), both countries have the same steady-state capital stock k and the same MPK. Therefore, the wage per efficiency unit in the two countries is equal. Workers, however, care about the wage per unit of labor, not the wage per efficiency unit. The wage per unit of labor is related to the wage per efficiency unit by the equation: wage per unit of L = wE. Thus, the wage per unit of labor is higher in the country with the more educated labor force.

10. Another growth model question.
a) You can calculate the MPK in two ways:
If you use calculus, $MPK = dY/dK = .3AK^{.3}\text{-}1L^{.7} = .3(AK^{.3}L^{.7})K^{-1}$
Substitute Y into the last expression for $AK^{.3}L^{.7}$ to get
$MPK = .3(Y)K^{-1} = .3(Y/K)$
Since we are told that the ratio of capital to output is 3, this makes Y/K = 1/3. Thus MPK = .3 (1/3) = 0.10
The second hint told you to make use of the definition of capital's share of output (see page 58). Thus capital's share = (MPK)(K/Y)
Capital's share of output is represented by the exponent of K in the production function, in this case, .3. Thus
0.3 = MPK (K/Y) = MPK (3) (since we know that the capital to output ratio is 3)
MPK = .3/3 = 0.10

b) In steady state, $\Delta K = sY - (\delta + n + g)K = 0$
or $sY = (\delta + n + g)K$
$s = (\delta + n + g)K/Y$
plugging in the values that we know:
s = (.04 + .03)3 = 0.21 [note that the growth of output equals n + g = 3%]

c) At the Golden Rule level of capital,
$MPK = (\delta + n + g) = 0.07$

d) From the second hint in part (a) above we know that
MPK = [capital's share](Y/K). Thus, at the golden rule,
MPK = .07 = [.3](Y/K)
Y/K = 0.233 or K/Y = 4.3

e) To find the saving rate necessary to achieve the Golden-Rule, use the equation for steady-state K (see part (b) above) and your answer to part (d): $s = (\delta + n + g)K/Y = (.04 + .03)(4.3) = 0.30$

11. Check your notes and the text.

12. This one is identical to #10 above except that the capital output ratio is 2.5 rather than 3.0.