

Learning About Population Growth

All of the environmental problems we have studied in the last two semesters are either caused or exacerbated by one common phenomenon: the increasing size of the human population. Every person on the planet requires certain environmental "services": oxygen, food, fresh water, shelter, and clothing. In order to provide these things for ourselves, we displace natural ecosystems to make way for housing, agriculture, transportation, and so on. These disturbances have become massive as we prospect and quarry for fuel and minerals to support our technological society. Technology has had an unwanted side effect of increasing the impact of each individual person. As more and more societies move towards higher levels of technology, the environmental problems caused by too many people will only worsen. Designing technologies which have minimal impact on the environment can reduce the impact each person has on the global ecosystem, but cannot solve these problems in the absence of a reduction in human populations. I hope by the time you are done reading this document you will come to realize that while technology exacerbates many environmental problems, the root cause lies in the growth of human populations.

This document was written to lead you through some of the mathematics underlying the study of population growth. It might go over your head in places, but be patient and keep reading. Try some of the problems, either by working them out on your calculator or by coming into the biology computer lab and using the program "ECOCYB", which can be accessed by double-clicking on the following icon:



Computers have made studying the growth of populations much simpler, and you should feel welcome to work your way through the examples contained in this document. Instructions on using ECOCYB are located at the end of this guide.

WHAT IS A POPULATION?

>a collection of organisms of the same species in some defined geographic area<

When we are discussing population growth, we are usually referring to the definition above. We may study and refer to the populations of squirrels on campus or the number of bluegills in a pond. Before getting too involved in any examination of a population's growth, be sure you understand which species is involved and what the geographic limits are. Often, the latter are quite arbitrary and may not be biologically significant.

Exponential Population Growth

I usually start a discussion of population growth by showing a video of the *Star Trek*TM episode *The Trouble with Tribbles* written by David Gerrold. Watching the show once, I was suddenly gripped with the urge to "check up" on Spock's calculation that there were 1,771,561 tribbles in a hold of grain. He was right! Given the assumptions that Spock rattles off (a few of which are irrelevant) there should have been 1,771,561 tribbles in the hold! David Gerrold had actually taken the time to do the rather simple calculations and come up with the right number, rather than picking it out of thin air as *Star Trek*TM writers usually do. And if a Hollywood screenwriter (on his first script) can understand it, so can you!

The tribbles exhibit what is known as EXPONENTIAL GROWTH. We'll come back to why it's named that shortly. Spock tells us the necessary parameters:

$$\text{birth rate} = 10 \quad \text{death rate} = 0 \quad \text{generation time} = 12 \text{ hrs}$$

This means that the average HERMAPHRODITIC (both male and female, from the Greek husband-and-wife gods Hermes and Aphrodite) tribble has a litter of ten babies every 12 hours, and that none of them died in the short time of the study.

We can make some simple calculations from Spock's data:

We start with one tribble (N_0), and after 12 hours:

$$12 \text{ hrs. } 1 \text{ tribble} \rightarrow 10 \text{ babies} + \text{original tribble} = 11 \text{ tribbles } (N_1)$$

12 hours later we have:

$$24 \text{ hrs. } 11 \text{ tribbles give birth to } 110 \text{ babies} = 110 + 11 \text{ parents} = 121 \text{ tribbles } (N_2)$$

... and so on for a three-day period:

$$36 \text{ hrs. } 121 \text{ tribbles give birth to } 1,210 \text{ babies} = 1,210 + 121 \text{ parents} = 1,331 \text{ tribbles } (N_3)$$

$$48 \text{ hrs. } 1,331 \text{ tribbles give birth to } 13,310 \text{ babies}$$

$$= 13,310 + 1,331 \text{ parents} = 14,641 \text{ tribbles } (N_4)$$

$$60 \text{ hrs. } 14,641 \text{ tribbles give birth to } 146,410 \text{ babies}$$

$$= 146,410 + 14,641 \text{ parents} = 161,051 \text{ tribbles } (N_5)$$

$$72 \text{ hrs. } 161,051 \text{ tribbles give birth to } 1,610,510 \text{ babies} \\ = 1,610,510 + 161,051 \text{ parents} = 1,771,561 \text{ tribbles!!! } (N_6)$$

Every 12 hours we have a new generation; note that each generation is 11x as big as the previous one, and that 11 is the sum of a parent tribble and its offspring. This allows us to construct a MODEL, a mathematical representation of the population growth of the tribbles:

$$r = \frac{b-d}{N}$$

r = growth rate or rate of natural increase

b = birth rate

d = death rate

N = population size

$$\Delta N = rN$$

(ΔN = change in population size)

For the tribble example, $r = (10 - 0)/1 = 10$

Question: If tribbles have 6 babies in an average litter, what would r equal?

To calculate population size, we multiply the rate of natural increase by the population size and add it to the population size:

$$N_{t+1} = rN_t + N_t \quad (\text{the "t" refers to time or generation})$$

From the tribble example:

$$N_2 = rN_1 + N_1 = 10 \times 11 + 11 = 110 + 11 = 121$$

This multiplying and adding process gets rather tedious. In this exponential growth model we can determine the future population size by taking the rate of population growth (r) and adding 1 to it to get a new value, R . The new value, R , can be raised to a power corresponding to the number of generations we are interested in. The general equation looks like this:

$$N_t = R^t N_0$$

The small t represents time (the number of generations), and N_0 is equal to the original population size. For tribbles, we can plug in the data to get this formula:

Tribbles: 6 generations:

$$R = r + 1 = 10 + 1 = 11$$

$$N_6 = R^6 \times 1 = 11^6 \times 1 = 1,771,561$$

Question: How many tribbles would there be in a eighth generation?

We can use this model (with different values) for other organisms as well. It also works well for human populations. Before we look at human populations, however, let us examine the general model in more detail.

Exponential population growth works just like interest accumulating in a bank account - tribbles have a high rate of interest! This becomes more obvious when the initial population size is larger than 1. For instance take this example with a population size of 1,000:

example: 30 births per 1,000 individuals, 20 deaths:

$$r = \frac{30 - 20}{1,000} = 0.01 = 1\% \text{ per year}$$

Question: If a population has a birth rate of 20 births per 100 individuals and a death rate of 50 per 1,000 individuals, what is r equal to?

Human populations are growing exponentially. This data is from the World Resources Institute:

$$\begin{aligned}
 1990: \quad & 27.1 \text{ births/1000 people; } B = 0.027 \\
 & 10 \text{ deaths/1000 people; } D = 0.010 \quad N = 5.292 \text{ billion} \\
 & r = (27.1 - 10)/1000 = 0.017 \\
 & R = 1 + 0.027 - 0.010 = 1.017
 \end{aligned}$$

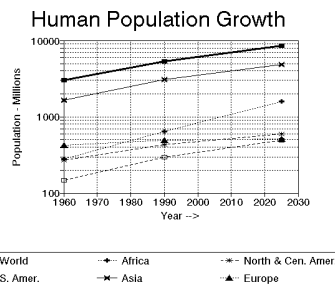
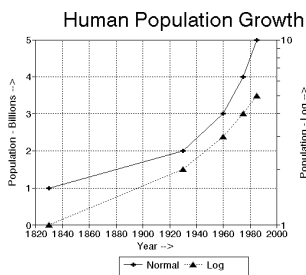
If we put these values into our model, we get the following results:

$$N_{1991} = 1.017 \times 5.292 = 5.382 \text{ billion; } \Delta N = 90 \text{ million}$$

$$N_{2025} = 1.017^{35} \times 5.382 = 9.709 \text{ billion; } \Delta N = 4.417 \text{ billion}$$

Question: How many people will there be in the year 2000?

The graph on the left below shows human population growth in the recent past. Note that it has both a NORMAL and a LOG plot (actually, a semi-log plot, since the data on the x-axis is plotted normally). The curved line of the normal plot is characteristic of exponential growth (the J-SHAPED CURVE), the straight line of the log plot (at least from 1930 on) **proves** that the growth is exponential. The graph on the right shows projected growth (using the model and the World Resources Institute Data; these log plots would hold if birthrates and death rates remain constant).



In nature, many populations grow exponentially - at least for a while. Eventually, however, the population growth rate levels off as either birth rates decrease or death rates increase. The changing birth and death rates are usually the result of overcrowding. As the organisms are packed together, they compete for food, water, shelter, etc. and may fall prey to epidemic diseases that spread faster under crowded conditions. A mother that is undernourished will not produce as many young, and an organism which is undernourished will not be able to survive adverse conditions - or escape predators. This decreasing population growth rate leads to our second population growth model - LOGISTIC GROWTH.

Logistic Population Growth

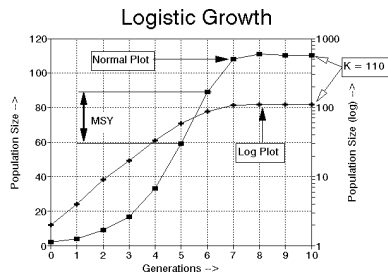
Remember from our model of exponential growth:

$$r = \frac{b-d}{N}$$

r = growth rate or rate of natural increase
 b = birth rate
 d = death rate
 N = population size

$$\Delta N = rN$$

Now, imagine what will happen if the birth rates fall and the death rates rise as the population gets larger. The value of r will grow closer and closer to zero, and the population will grow less and less with each successive generation until it reaches a steady-state. The population level at this steady state is known as the **CARRYING CAPACITY** or K .



The graph to the left shows an example of logistic growth representing bass in a farm pond over 10 generations (10 years). Note the 'S' shape of the curve; this is typical of the logistic growth model. The log plot shows an initial burst of nearly **EXPONENTIAL GROWTH**, followed by a decline in the growth rate as the population size increases and the density dependent factors begin to reduce the rate of population increase. The maximum increase in population from one generation to the next occurs at the middle of the curve at

generation 5 and is an important management tool known as the **MAXIMUM SUSTAINABLE YIELD (MSY)**; for this pond it is equal to 30. In theory, it is possible to remove (by fishing) 30 bass each year if the population is not allowed to fall below 59 bass. At this point, the population is almost doubling each year. Below this point, the population is doubling, but there aren't as many fish to double, and above this point, density dependence means that the birth rate drops and the population no longer doubles. The population levels off at about 110, this is the **CARRYING CAPACITY** of the pond, which we designated K .

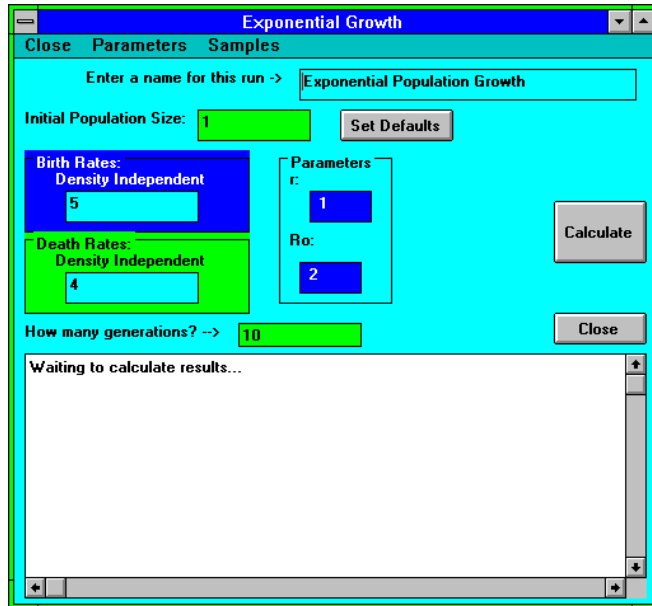
Many populations exhibit this logistic growth pattern. Note that in its early stages logistic growth resembles exponential growth; the differences become apparent only when a population growing logistically reach about 1/2 of their carrying capacity.

The actual calculations for calculating logistic growth, while simple, are more involved than we need to go into in Leadership 333. Still, you can experiment with the values by using the computer program.

Experimenting with population growth - the Ecocyb program

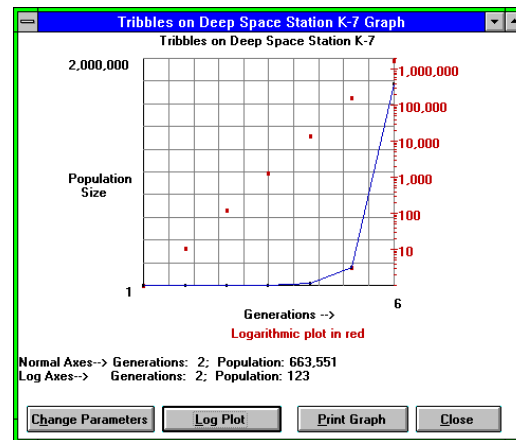
When the program first comes up, you are given the choice of running several programs. For our purposes, we will concentrate on two of them - exponential population growth and logistic population growth. Start the exponential population growth program by double-clicking on its button. The screen will look like this:

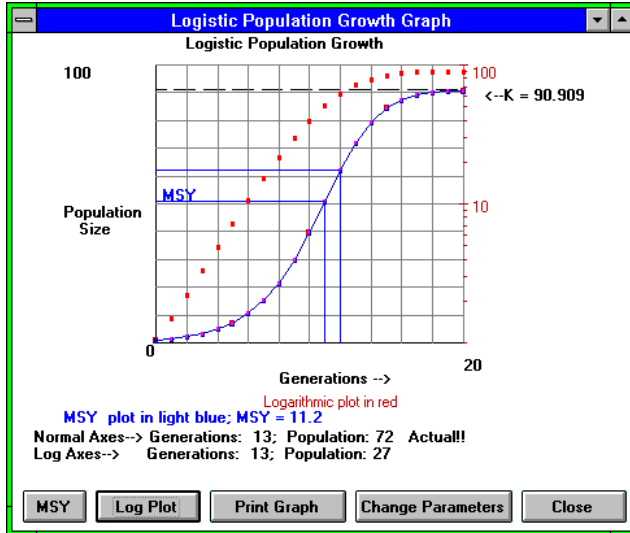
You can change the values for the title, the initial population size (N_0), the birth rate, the death rate, and the number of generations. When the values are set the way you want, click on the calculate button to calculate the results. Note that you can instantly set the parameters to give you results similar to those in your textbook by clicking on the samples choice from the menu at the top of the screen. There are several choices there, among them are one for Tribbles™ and one for Mader.



Once the calculations have been made, a button appears to allow you to graph the data (if you change any parameters, the button disappears and you cannot graph until you recalculate). The graph screen looks like this:

Feel free to experiment with different values and see what effect they have on growth. Note that exponential growth is indicated by the log plot falling in a straight line. If you are not sure where a point falls, move the mouse pointer (it changes to a crosshairs in the graph) over the point. The program will show you the *approximate* value in the lower left portion of the screen. If you move the pointer over the actual plotted point for a value, you will be rewarded with the "actual value"; but be careful that you have positioned the cursor properly. If you move it slightly and the "actual" value changes, watch the generation value and be sure you are getting the "actual" value for the right generation!





The logistic growth program works in a similar fashion to the exponential growth program; but there are two additional parameters, density dependent birth and death rates, to deal with. These new parameters measure the effects of increasing population on the birth rate and death rates; the larger the population is the more important these parameters become. The graph for the logistic growth program is slightly different from the exponential growth graph. In addition to the normal and log plots, the logistic growth graph can also point out the values associated with the maximum sustainable yield. If you look at the graph to the left, you will see a set of four lines (these are light blue on the screen). The

two horizontal lines indicate the upper and lower bounds of the population levels at the maximum sustainable yield; the MSY is equal to the difference between these values. The vertical lines indicate the generations at which the MSY will be reached. Note also that the K value is indicated by a dashed line.

This program has many options under the samples; most of them deal with showing how increasing the growth rate of the population can lead to chaotic behavior. Chaos is one of the cutting edges of biological theory today, and it grew out of a study of this model. Some of the graphs are pretty bizarre. Change the parameters as you see fit. Have fun. Experiment by changing one value at a time in a systematic way until you understand the effect each parameter has on population growth.

A final check of your skills:

A reference of particular use to environmentalists is the annual compilation *World Resources*, produced by the World Resources Institute. Combined with the programs, the data provided by *World Resources* can help you predict future trends. To test your skills, go to library and obtain data from *World Resources* on crude birth and death rates, as well as population levels in 1995, for an African country of your choice. What will its population be in 2025 according to our model? According to the World Resources Institute? Why is there a difference?

(answer on next page - no peeking!)

Using Burkina Faso as an example:

1995 population: 10.4 million
Birth Rate: 47 births/1000 individuals Death Rate: 17
deaths/1000 individuals
Projected 2023 population: 23.71 million

Using the exponential growth model:

Given these initial conditions:

Initial population size --> 10400000
Density independent birth rate (Bo) --> .047
Density independent death rate (Do) --> .017
Intrinsic rate of increase (r) --> .03
Initial population growth rate (Ro) --> 1.03

Generation: 1
Total Population Size = 10,712,000. R = 1.03
Delta N = 312,000. Delta N/N = .03

Generation: 30
Total Population Size = 25,243,512. R = 1.03
Delta N = 735,248. Delta N/N = .03

The final population size as predicted by our model is 25,243,512; the WRI estimate is 23.71 million, a difference of 1,533,512. The difference is most likely due to the WRI's use of a demographic model rather than the simple exponential growth model.