Section 2.4 - Measures of Variation

1. Example #1: (p. 84, pr. 2)) Consider the population data set
   13 23 15 13 18 13 15 14 20 20 18 17 20 13

2. Definitions: (Find for above example.)

   (a) *Range* = difference between the maximum and minimum data entries.

   (b) *Deviation* = difference between an entry x and the mean μ.

   (c) *Population variance* $= \sigma^2 = \dfrac{\sum(x-\mu)^2}{N}$, where $N = \#$ of entries.

   (d) *Population standard deviation* $= \sigma = \sqrt{\sigma^2} = \sqrt{\dfrac{\sum(x-\mu)^2}{N}}$.

3. Example #2: Consider the following sample of the first seven entries from the above population.

4. Definitions: (Find for the above example.)

   (a) *Sample variance* $= s^2 = \dfrac{\sum(x-\bar{x})^2}{n-1}$, where n = sample size.

   (b) *Sample standard deviation* $= s = \sqrt{s^2} = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$.

5. Empirical Rule (68-95-99.7 Rule) For data with a (symmetric) bell-shaped distribution, the s.d. satisfies the following.

   (a) About 68% of the data lie within one s.d. of the mean.

   (b) About 95% of the data lie within two s.d. of the mean.

   (c) About 99.7% of the data lie within three s.d. of the mean.

6. Example #3: (p. 88, pr. 30)
   The mean value of land and buildings per acre from a sample of farms is \$1200, with a standard deviation of \$350. Between what two values do about 95% of the data lie? What about 68%, 99.7%? (Assume the data set has a bell-shaped distribution.)

7. Chebychev's Theorem:
   The proportion of any data set lying within k standard deviations ($k > 1$) of the mean is at least $1 - \frac{1}{k^2}$.

   - $k = 2$: In **any** data set, at least $1 - \frac{1}{2^2} = \frac{3}{4}$ or 75% of the data lie within two s.d. of the mean.
   - $k = 3$: In **any** data set, at least $1 - \frac{1}{3^2} = \frac{8}{9}$ or 88.9% of the data lie within three s.d. of the mean.

8. Example #4: (p. 88, pr. 36)
   The mean time in a women's 400-meter dash is 52.37 seconds, with a standard deviation of 2.15. Apply Chebychev's Theorem to the data using $k = 2$. Interpret the results.