

Section 3.1 - Basic Concepts of Probability

1. Definitions:

- **Chance Behavior** or **Random phenomenon** - has outcomes that we cannot predict but that nonetheless have regular distribution in very many repetitions.
- **Probability** - the chance something happens (the proportion of times that a particular outcome would occur in a long series of repetitions)
- **Probability Experiment** - any experiment that has two or more outcomes; e.g. Tossing a coin, Drawing a card, Rolling a die
- **Outcome** - the result of a single trial in a probability experiment.
- **Sample Space** (S) - the set of all possible outcomes of a probability experiment.
- **Event** - any collection of outcomes (a subset of the sample space)
- An event that consists of a single outcome is called a **simple event**.

2. Example #1: **Sample Space**

Choose a student at random from a large statistics class. Describe a sample space S for each of the following. (May be more than one answer.)

- (a) Ask how much time the student spent studying during the past 24 hours.
- (b) Ask how much money in coins (not bills) the student is carrying.
- (c) Record the student's letter grade at the end of the course.
- (d) Ask whether the student did or did not take a math class in each of the two previous years of school.

3. **Probability Model** - a mathematical description of a random "experiment." The model contains two parts: the sample space, S , and the method for assigning probability to each event.

4. **Independent trials** - the outcome of one trial does not influence the outcome of another trial

5. Three types of probability:

- **Classical probability**: each outcome in a sample space is equally likely to occur.

$$P(E) = \frac{\# \text{ of outcomes in } E}{\text{Total } \# \text{ of outcomes in sample space}}$$

- Examples: Rolling a die, flipping a coin, etc.
- **Empirical probability**: based on observations obtained from probability experiments.

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}$$

- Examples: An insurance company determines that in every 100 claims, 4 are fraudulent. What is the probability that the next claim the company processes will be fraudulent?

- **Subjective probability:** result from intuition, educated guesses, and estimates.
- Examples: Probability of a given team winning the Super Bowl, chance for recovery from an illness, etc.

6. **Law of Large Numbers:**

As an experiment is repeated over and over, the empirical probability of an even approaches the theoretical probability of an event. (In the end, the house always wins.)

7. Example: Flipping a coin.

8. Probability Rules:

- (a) If A is an event, then the probability of A must satisfy $0 \leq P(A) \leq 1$.
- (b) If S is the sample space, then $P(S) = 1$.
- (c) For any event A , the probability that A does not occur is $P(A') = 1 - P(A)$.
- (d) If A and B are events that have nothing in common (**disjoint** events), then $P(A \cup B) = P(A) + P(B)$ where $(A \cup B)$ means "A or B (or both) occurs".

9. Example #2: (p. 126, pr. 16) Consider a company that selects employees for random drug tests. The company uses a computer to select randomly employee numbers that range from 1 to 6,296. Find the probability of selecting a number that is not divisible by 1000.

10. Example #3: (p. 126, pr. 18, 20, 24)