Section 4.1 - Probability Distributions

1. **Random variable** (x) - represents a numerical value associated with each outcome of a random experiment (Usually denoted by capital letters near the end of the alphabet, such as X or Y.)

   - **Discrete Random Variable** - has a finite list of possible outcomes or $\{0, 1, 2, \ldots\}$
   - **Continuous Random Variable** - can take on any value in an interval on the number line

   Example: Athletics: Number of tackles made vs. 100m. dash time (decimals allowed).

2. Example #1: (p. 179, pr. 12, 14, 16) Determine whether the random variable x is discrete or continuous.

   (a) x represents the length of time it takes to get to work.

   (b) x represents the number of rainy days in the month of July in Florida.

   (c) x represents the weight of a chemical compound (in pounds).

3. **Probability Distribution** - similar to a probability model.
   A probability distribution (or a way to assign a probability to each value or each interval).

4. **Discrete Probability Distribution:** gives all possible values of a random variable and the corresponding probability for each value. Must satisfy the following:

   (a) $0 \leq P(x) \leq 1$ (Say in words)

   (b) $\sum P(x) = 1$ (Say in words)

5. Since probabilities represent relative frequencies, a discrete probability distribution can be graphed with a relative frequency histogram.

6. Constructing a Discrete Probability Distribution: Assume x has possible outcomes $x_1, x_2, \ldots x_n$

   (a) Make a frequency distribution for all possible outcomes.

   (b) Find the sum of the frequencies.

   (c) Find each probability by dividing frequency by sum of frequencies.

   (d) Check that each probability is between 0 and 1 and sum to 1.

7. Example #2: (p. 181, pr. 24) Decide whether the distribution is a probability distribution.

x	0	1	2	3
$P(x)$	0.005	0.435	0.555	0.206

8. Example #3: (p. 181, pr. 28) Find probability distribution, mean, variance, s.d., interpret
   The number of cats per household in a small town.

Cats	0	1	2	3	4	5
Households	1941	349	203	78	57	40

9. The **mean** of a discrete random variable is given by

$$\mu = \sum xP(x).$$

   The mean represents the 'theoretical' average - may not be a possible outcome.
   Law of large numbers says that if the experiment were performed many times, the mean of all outcomes would be close to the mean of the random variable.

10. The **variance** of a discrete random variable is

$$\sigma^2 = \sum (x - \mu)^2 P(x).$$

11. The **standard deviation** is

$$\sigma = \sqrt{\sigma^2}.$$

   Interpretation: "Most" (if bell shaped, 68%) outcomes differ from the mean by at most 1 s.d.

12. The **expected value** of a discrete random variable is equal to the mean of the random variable.

$$E(x) = \mu = \sum xP(x).$$

   Interpretation: What you should "expect" your outcome to be.