Section 6.3 - Confidence Intervals for Populations Proportions

1. Definition:
   The point estimate for p, the population proportion of successes, is given by the proportion of successes in a sample $\hat{p} = \frac{x}{n}$. The point estimate for the proportion of failures is $\hat{q} = 1 - \hat{p}$.

2. Example #1: A study of 2008 traffic fatalities found that 800 were alcohol related. Find $\hat{p}$ and $\hat{q}$.

3. Central Limit Theorem for Sample Proportions: (categorical data)
   Suppose we take an SRS of size n from a population that contains a proportion p of successes. Let $\hat{p}$ be the sample proportion of successes. If the sample is large enough (i.e. if $n\hat{p} \geq 5$, $n\hat{q} \geq 5$), then

   (a) The sampling distribution of $\hat{p}$ is approximately normal.
   (As n increases, sampling distribution of $\hat{p}$ becomes more normal. Don't use for small n.)

   (b) The mean of the sampling distribution is $\mu_{\hat{p}} = p$.

   (c) The standard deviation of the sampling distribution is $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$.

4. Confidence Intervals for Sample Proportions:
   Choose an SRS of size n (where n is sufficiently large) from a large population that contains an unknown proportion p of successes. Call the proportion of successes in this sample $\hat{p}$. An approximate 95% confidence interval for the parameter p is $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$.
   (Observe that the standard deviation is only dependent on $\hat{p}$ and n, not σ.)
   Similarly, a 90% C.I. is $\hat{p} \pm 1.645\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$ and a 99% C.I. is $\hat{p} \pm 2.58\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$.
   In general, the confidence interval is $\hat{p} \pm z_c\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$. (**Standard error** of $\hat{p}$.)

5. Example #1:
   A department store decides to examine the proportion of shoppers interested in a boutique format for the store's basement. Of the 250 shoppers surveyed, 175 think it would be a good idea.

   (a) Say in words what the population proportion p is for this situation.

   (b) Find a 95% confidence interval for the proportion of all shoppers who approve of the boutique format. Be sure to show ALL relevant work.

   (c) Interpret the resulting interval in words that a statistically naïve reader would understand.

   (d) Given your interval in part (c), can you conclude that the majority of all shoppers would approve of the boutique format? Why or why not?

6. Example #2:
   We have all heard that 98.6 degrees Fahrenheit is "normal body temperature". In fact, there is evidence that most people have a slightly lower body temperature. You plan to measure the body temperature of a random sample of people very accurately. You hope to show that a **majority** have temperatures lower than 98.6 degrees. Describe the population proportion p in this setting.

7. Example #3:
   A survey show that 51% of adults prefer to receive advertisements through the mail. The m.o.e. E is 5.2%. Construct a confidence interval for the proportion of adults who prefer to receive advertisements through the mail.