Section 7.2 - Hypothesis Testing for the Mean (Large Samples)

1. Steps for Hypothesis Testing

   (a) Determine the hypotheses.

   - **Null Hypothesis**: an assumption concerning the value of the population parameter being studied (usually represents no effect, no change, no difference, etc.; $\leq, \geq, =$)
     Notation: H_0
   - **Alternative Hypothesis**: the complement of the null hypothesis (must be true if H_0 is false). (states the result for which we hope to find evidence; $<, >, \neq$)
     Notation: H_a or H_A

   Note: The null hypothesis may or may not be true. We will carry out a study and then determine if we have strong enough evidence to conclude that the null hypothesis is false (meaning our evidence suggests that H_A is true).

   (b) Specify the level of significance (α): how "strong" of evidence we'll require to reject H_0.

   (c) Obtain a simple random sample of n observations from the desired population and calculate the observed sample statistic. (This is referred to as the **test statistic**.)

   For example, if we want to test something about a population mean (μ), then we would calculate the sample mean ($\bar{x}$). If we want to test something about a population proportion (p), then we would calculate the sample proportion ($\hat{p}$).

   (d) Determine the "strength" of your evidence.
   The evidence is **strong** if the outcome we observe would rarely occur assuming the null hypothesis is true (meaning it is more probable that the alternative hypothesis is true).

   The evidence is **weak** if the outcome we observe has a high probability of occurring assuming the null hypothesis is true.

   We measure the strength of the evidence by calculating a P-value.

   P-value: the probability of obtaining a sample outcome as extreme or more extreme than the actual observed outcome, *assuming that the null is true.*

   - Smaller P-value $\Rightarrow$ stronger evidence is against H_0.
   - P-value describes the risk of making a mistake if we wrongly reject the null.

   (e) Draw a conclusion.

   - If the P-value is "small" ($P \leq \alpha$), then we reject H_0 in favor of H_A.
   - If the P-value is "large" ($P > \alpha$), then we fail to reject H_0, meaning we cannot conclude H_A.

   Note: You may NEVER conclude that the null is true.
   Unfortunately, you CANNOT be certain that you have made the correct conclusion.

   (f) Interpret your decision and state your P-value.

2. Section 7.1 (Introduction to Hypothesis Testing)
   - Determining H_0 and H_a.
   - Possible Errors with Hypothesis Testing: Type I and Type II.
   - Types of Hypothesis Tests: left-tailed, right-tailed, and two-tailed.
   - Decisions Based on P-value: If $P \leq \alpha$, then reject H_0; If $P > \alpha$, then fail to reject.
3. Section 7.2 (Hypothesis Testing for the Mean: Large Samples)
   - Finding the P-value.
   - z-test for a mean: When $n \geq 30$ or population is normal and σ is known.
   - Determining critical values (z_c) for the rejection region.
4. Example #1: Finding the P-value. (Also determine whether or not to reject H_0.)
   (a) Left-tailed test, $z = -2.05$; $\alpha = 0.05$
   (b) Right-tailed test, $z = -2.52$; $\alpha = 0.01$
   (c) Two-tailed test, $z = -1.93$; $\alpha = 0.05$
5. z-Test for a Mean μ: Can be used when $n \geq 30$ or when the population is normal and σ is known.
   $$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}.$$
   As with confidence intervals, if $n \geq 30$, you can use s in place of σ.
6. Examples: Determining a P-value
   Go through Chapter 7: Examples #5, #6, and #7.
7. Rejection region and critical values (z_c): z_c separates rejection region from nonrejection region.
   (a) left-tailed: find the z-score that corresponds to an area of α.
   (b) right-tailed: find the z-score that corresponds to an area of $1 - \alpha$.
   (c) two-tailed: find the z-score that correspond to an area of $\frac{1}{2}\alpha$ **and** $1 - \frac{1}{2}\alpha$.
8. Example #2: Find the critical value(s) and rejection region if $\alpha = 0.01$ for
   (a) a left-tailed hypothesis test.
   (b) a right-tailed hypothesis test.
   (c) a two-tailed hypothesis test.
9. Critical values for common α's:

Alpha (α)	0.10	0.05	0.01
left-tailed	-1.280	-1.645	-2.330
right-tailed	1.280	1.645	2.330
two-tailed	$\pm$ 1.645	$\pm$ 1.960	$\pm$ 2.576