Exam 3 Name:_____

Math 125.01
April 26, 2007

| **Question** | **Points Earned** | **Points Possible** |
|---|---|---|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 10 |
| 9 | | 10 |
| | | 10 |
| Bonus | | |
| Total | | 100 |

Math 125, Section 01 Name:_____
Exam 3 - April 26, 2007

1. Given the function $f(x) = x^3 - 6x^2 - 36x + 48$, determine the following:

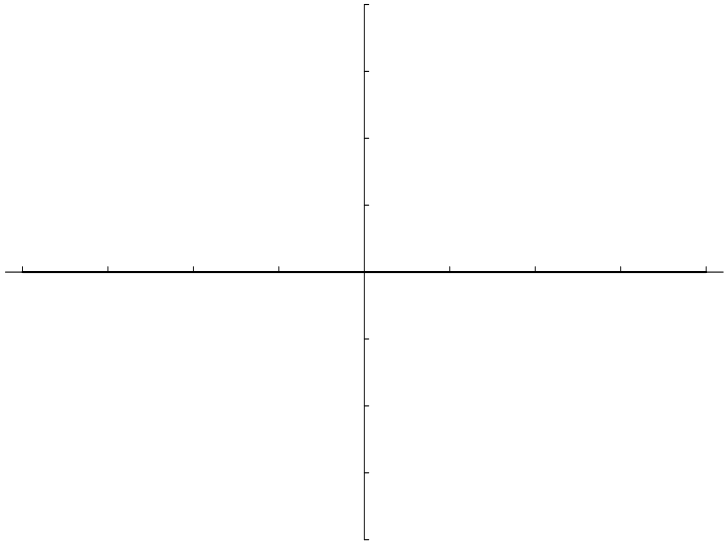
   (a) The intervals on which $f(x)$ is increasing those on which $f(x)$ is decreasing.

   (b) Local extreme points (Label which are local max's and which are local min's).

   (c) The intervals on which the graph is concave upward/downward.

   (d) Point(s) of inflection.

   (e) Sketch the curve. (Be sure to scale your axes or label important points)



2. Given the function $f(x) = \dfrac{-1}{x^2 - x - 6}$, we can calculate that

$$f'(x) = \frac{2x - 1}{(x^2 - x - 6)^2} \qquad \text{and} \qquad f''(x) = \frac{2(3x^2 - 3x + 7)}{(x^2 - x - 6)^3}$$

Determine the following:

(a) The intervals on which $f(x)$ is increasing those on which $f(x)$ is decreasing.

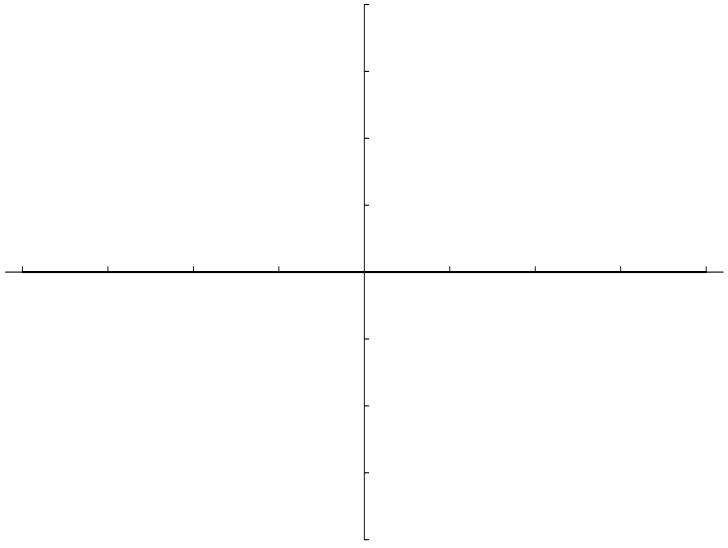
(b) Local extreme points (Label which are local max's and which are local min's).

(c) The intervals on which the graph is concave upward/downward.

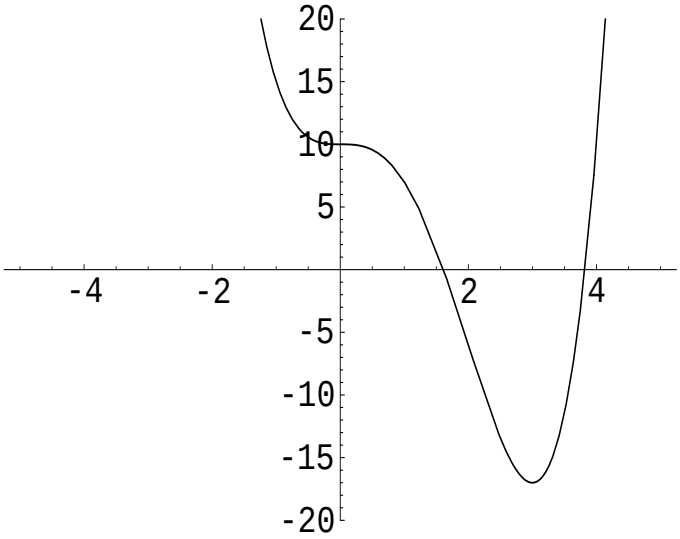
(d) Point(s) of inflection.

(e) Asymptotes (horizontal and vertical) and intercepts ($x$-intercepts and $y$-intercepts)

(f) Sketch the curve. (Be sure to scale your axes or label important points)



3. Using the graph of $f(x)$ below, determine if $f(x)$, $f'(x)$, and $f''(x)$ are positive, negative, or zero at each marked point.




| | $f(x)$ | $f'(x)$ | $f''(x)$ |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |
| D | | | |
| E | | | |

4. State the Mean Value Theorem.
(Be sure to include both the necessary hypotheses and the conclusion.)

5. Find $f(x)$ in each of the following:

   (a) $f''(x) = \sin x$.

   (b) $f''(x) = x + \sqrt{x}$, $f(0) = 1$, $f'(0) = 2$.

6. Find the most general antiderivative of the following.

(a) $\int \left(3\sqrt{t} + \frac{4}{t^2}\right)\, dt$

(b) $\int \frac{3}{x} + e^{3x} + \sec^2(5x)\, dx$

7. Find the following limits.

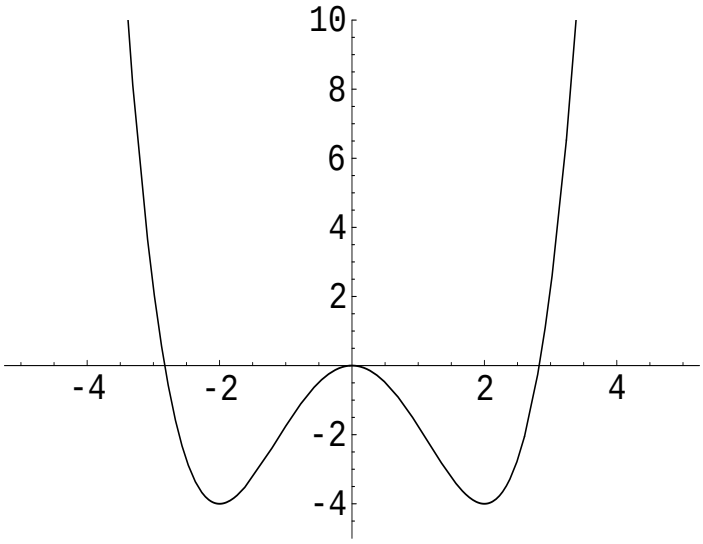
(a) $\lim_{x\to\infty} x^2 e^{-x}$

(b) $\lim_{x\to 0} \frac{4 - 4e^x}{xe^x}$

(c) $\lim_{x\to\frac{\pi}{2}} \frac{\sin 2x}{1 - \cos 2x}$

(d) $\lim_{x\to 0} \frac{5 - 5\cos x}{e^x - x - 1}$

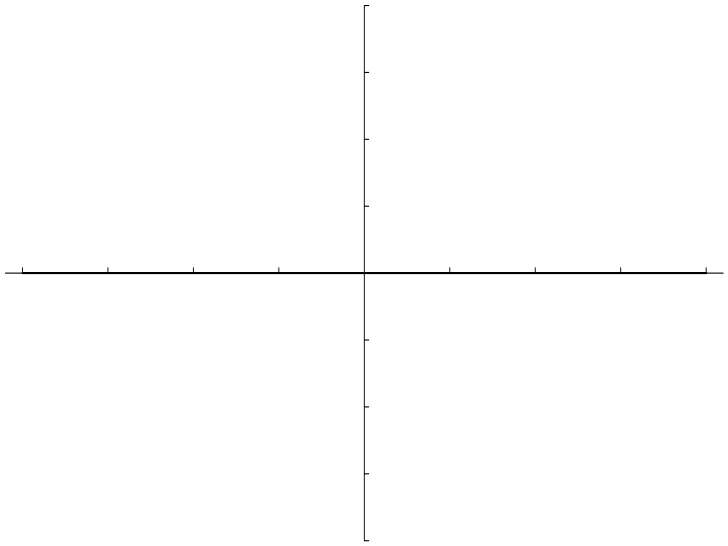
8. The figure below shows the graph of the derivative, $f'(x)$, of a function $f(x)$.




(a) On what intervals is $f(x)$ increasing or decreasing?

(b) On what intervals is $f(x)$ concave upward or concave downward?

(c) Sketch a possible graph of $f(x)$.



9. Answer **2** of the following **3** optimization problems. If you do not **clearly** mark which **2** that you wish to have graded, I will grade the first 2.

   (a) A box with a square lid and open top must have a volume of $32\text{ft}^3$. Find the dimensions of the box that minimizes the amount of material used.

   (b) You are designing a rectangular poster. You need to have a 4-in margin at the top and bottom and a 2-in margin at each side. What dimensions for the print area will maximize the amount of print you can get out of a sheet with an overall area of 162 $\text{in}^2$?

(c) An 864 $m^2$ rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

Bonus For what values of the constants $a$ and $b$ is (2,-3) a point of inflection of the function $f(x) = ax^3 - x^2 + b$?