Math 125.01 Name:______________________________

Lab 4 - Sections 2.4-2.7

February 12, 2008

1. Determine the following limits.

   (a) $\lim_{x \to 0} 6x^2(\cot x)(\csc 2x)$

   (b) $\lim_{x \to 0} \dfrac{\sin 5x}{\sin 4x}$

2. For each part, sketch a graph of a function which satisfies **all** of the properties listed: (You should have one sketch for part (a) and one for part (b).)

   (a) $\lim_{x \to 0} f(x) = 0,\ f(0) = 3,\ \lim_{x \to 1^+} f(x) = -1,\ \lim_{x \to 1^-} f(x) = 1,\ f(1) = 0.$

   (b) $\lim_{x \to 2^+} f(x) = \infty,\ \lim_{x \to 2^-} f(x) = -\infty,\ \lim_{x \to 0} f(x) = -\infty,\ \lim_{x \to -\infty} f(x) = \infty,\ \lim_{x \to \infty} f(x) = 0.$

3. From the graph of $g$, state the intevals on which $g$ is continuous.

4. (a) Graph $f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1. \end{cases}$

   (b) Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

   (c) Does $\lim_{x \to 1} f(x)$ exist? If so, what is it? If not, why not?

   (d) Is $f(x)$ continuous at $x = 1$? Why or why not?

5. Let $f$ and $g$ be functions which are discontinuous at $c$. Give examples to show the following.

   (a) $f + g$ can be continuous at $c$.

   (b) $f + g$ can be discontinuous at $c$.

   (c) $fg$ can be continuous at $c$.

   (d) $fg$ can be discontinuous at $c$.

6. For each of the following functions, **explain** where the function is continuous, continuous from the right, continuous from the left, and discontinuous. Also, sketch the graph of each function.

   (a) $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ \frac{1}{x^2} & \text{if } x > 1 \end{cases}$

   (b) $g(x) = \begin{cases} -\frac{1}{(x-1)^2} & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ \frac{x^2-1}{x+1} & \text{if } x > 1 \end{cases}$

7. The **Intermediate Value Theorem** says:

   If $f$ is continuous on the closed interval $[a, b]$ and $k$ is any number between $f(a)$ and $f(b)$,then there is a least one number $c$ in $[a, b]$ such that $f(c) = k$.

   Use the Intermediate Value Theorem to show that $f(x) = x^3 + 2x - 1$ has a zero in the interval $[0, 1]$.