Math 125 - Exam 1 Practice Test

1. Solve the following equation for x.

$$\ln e + e^{\ln 2 + \ln x} = \frac{1}{x} \ln e^3.$$

2. Find the following limits.

   (a) $\displaystyle\lim_{x \to -3} \frac{x^2 - x - 12}{x + 3}$

   (b) $\displaystyle\lim_{x \to 2} \frac{x^2 - 4x + 3}{x + 2}$

   (c) $\displaystyle\lim_{x \to -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}$

   (d) $\displaystyle\lim_{x \to 0} \frac{2 \sin 5x}{3x}$

3. If $\sqrt{3 - 2x^2} \leq f(x) \leq \sqrt{3 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \to 0} f(x)$.

4. Suppose $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ both exist. Suppose also that $\lim_{x \to c}[f(x) + g(x)] = 12$ and that $\lim_{x \to c}[f(x)g(x)] = 27$.

   (a) Find $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$, assuming $\lim_{x \to c} f(x) \leq \lim_{x \to c} g(x)$.

   (b) Find $\lim_{x \to c}[f(x) - g(x)]$.

   (c) Find $\displaystyle\lim_{x \to c} \frac{f(x)}{g(x)}$.

5. Determine if the following functions have any points of discontinuity. For any such points, label the discontinuities as removable or non-removable.

   (a) $f(x) = \dfrac{x + 2}{x - 5}$ (b) $f(x) = \dfrac{x - 1}{x^2 - 1}$ (c) $f(x) = 7x - \dfrac{4}{x + 6}$

6. For what value of a is the following function continuous at every x?

$$f(x) = \begin{cases} x^2 - 3 & x > 3 \\ ax + 5 & x \leq 3 \end{cases}$$

7. Find the value of the constant k such that the limit below exists. Then, find the limit.

$$\lim_{x \to 5} \frac{x^2 - 7x + k}{x - 5}$$

8. Let $f(x) = 5x^2 - 3x$.

   (a) Find the slope of the function's graph at the point (2,14).

   (b) Find the equation for the line tangent to $f(x)$ at (2,14).

9. Let $f(x)$ be the function graphed below. Use this graph to find the following.

(a) $\lim_{x\to -2^+} f(x)$
(b) $\lim_{x\to 0^-} f(x)$
(c) $\lim_{x\to 2} f(x)$
(d) $\lim_{x\to -3} f(x)$
(e) $\lim_{x\to \infty} f(x)$
(f) $f(-2)$
(g) $f(2)$
(h) At what points is $f(x)$ discontinuous?
(i) Which discontinuity point(s) is (are) removable? How could we fix the discontinuity?
(j) For which values of x_0 does $\lim_{x\to x_0} f(x)$ not exist?
(k) Is $f(x)$ continuous on the interval [1,4]? Why or why not?

10. Find the following limits, letting $f(x) = \dfrac{x^2 - 3x + 2}{x^3 - 4x}$.

(a) $\lim_{x\to 1} f(x)$
(b) $\lim_{x\to 0^-} f(x)$
(c) $\lim_{x\to -2^+} f(x)$
(d) $\lim_{x\to 2} f(x)$
(e) $\lim_{x\to -\infty} f(x)$

11. Determine if the following statements are true or false. If the statement is false, sketch a graph that contradicts the statement.

(a) If a function is differentiable on an interval, then it is continuous on the interval.
(b) If a function is continuous on an interval, then it is differentiable on the interval.
(c) If $\lim_{x\to 3} f(x)$ exists, then $f(x)$ is continuous at $x = 3$.

12. Find $\dfrac{dv}{dt}$ if $v = \dfrac{1}{t} - 2t$ using the definition of derivative. (You can't use power rule.)

13. Find $f'(x)$ for the following functions.

(a) $f(x) = 7x^3 - 6x^{2/3} - \dfrac{5}{x^4} + e^{-x} + 5\pi$
(b) $f(x) = 2x\cos x + x^2 e^x$
(c) $f(x) = (x^3 - 5\sqrt{x})(4x^2 + 3x^8 + e^{5x})$
(d) $f(x) = \dfrac{t^2 - 1}{t^2 + 1}$