Math 125 - Practice Exam 2

1. Find $f'(x)$ for the following functions.

   (a) $f(x) = (2x^4 + \frac{1}{\sqrt{x}})(-9x - e^{x^2} + 13)$

   (b) $f(x) = \dfrac{x^6 - x}{2 - \cos(3x)}$

   (c) $f(x) = \sqrt[5]{(x^2 + 6x)^3}$

   (d) $f(x) = \sin(\cos(7x^3))$

2. Find $f'(x)$ for the following functions.

   (a) $f(x) = 7\ln(x^8 + 2x)$

   (b) $f(x) = \ln\left(\dfrac{(x^2+3)^8}{\sqrt{1-x}}\right)$

   (c) $f(x) = (\ln(x^5 - x))^3$

3. Find $f'(x)$ if $f(x) = e^{\cos(7x)} + \sin^5(x + e^x + x^5)$.

4. Calculate $f''(x)$ if $f(x) = \dfrac{4}{\sqrt{3x - 21}}$.

5. Prove that if $f(x) = \cot x$, then $f'(x) = -\csc^2 x$.
   (You may use known derivatives of $\sin x$ and $\cos x$.)

6. Find $f'(x)$ if $f(x) = \cos^{-1}(2x + 1)$

7. At time $t \geq 0$ (t in seconds), the position of a body moving along the s-axis is $s(t) = t^3 - 6t^2 + 9t$ m.

   (a) Find the body's acceleration each time the velocity is zero.

   (b) Find the body's speed each time the acceleration is zero.

   (c) When is the body moving forward? Backward?

8. Find $f'(x)$ if $f(x) = x^3 \tan^{-1}\sqrt{x^2 + 1}$

9. The total surface area S of a right circular cylinder is related to the base radius r and height h by the equation $S = 2\pi r^2 + 2\pi rh$.

   (a) How is dS/dt related to dr/dt if h is constant?

   (b) How is dS/dt related to dr/dt and dh/dt if neither r nor h is constant?

10. Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = -1$ and $x = 5$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	5	11	-1/3	1
5	4	3	2	-7

Find the derivatives below with respect to x at the given values.
(Simplified answers are expected.)

(a) $4f(x) + 9g(x)$, $x = -1$

(b) $\sqrt{f(x)}g(x)$, $x = 5$

(c) $g(f(x))$, $x = -1$

(d) $(x^3 + g(x))^2$, $x = -1$

11. If $x^2 - 5x^4y^2 - y^3 = 1$, then find $\dfrac{dy}{dx}$.

12. Find the equation of the tangent line to the curve $xy + 2x - 5y = 2$ at the point $(3, 2)$.

13. The volume of a cube is increasing at the rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the lengths of the edges changing at that instant?

14. Find the derivative for each of the following functions.

(a) $y = \tan^{-1}(e^{-t})$

(b) $f(x) = \tan^{-1}\sqrt{x^2 - 1} + \sin^{-1}(3x)$

15. Use the linearization of $f(x) = e^{-x}$ to approximate $f(-0.1)$.