

Exam 1

Name: _____

Math 125.01
February 19, 2008

Question	Points Earned	Points Possible
1		16
2		12
3		5
4		5
5		6
6		6
7		5
8		16
9		10
10		10
11		4
12		5
Total		100

1. Find the following limits.

(a) $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5} - 2}{x - 3}$

(b) $\lim_{x \rightarrow -2} \frac{x^2 + 4x + 4}{x - 2}$

(c) $\lim_{x \rightarrow -2} \frac{3x + 4}{x^2 - 4x - 12}$

(d) $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x^2 - 9x + 20}$

2. Find the following limits, letting $g(x) = \frac{2x^3 - 4x^2 - 16x}{x^2 - 4}$.

(a) $\lim_{x \rightarrow -2} g(x)$

(b) $\lim_{x \rightarrow 2^-} g(x)$

(d) $\lim_{x \rightarrow 2^+} g(x)$

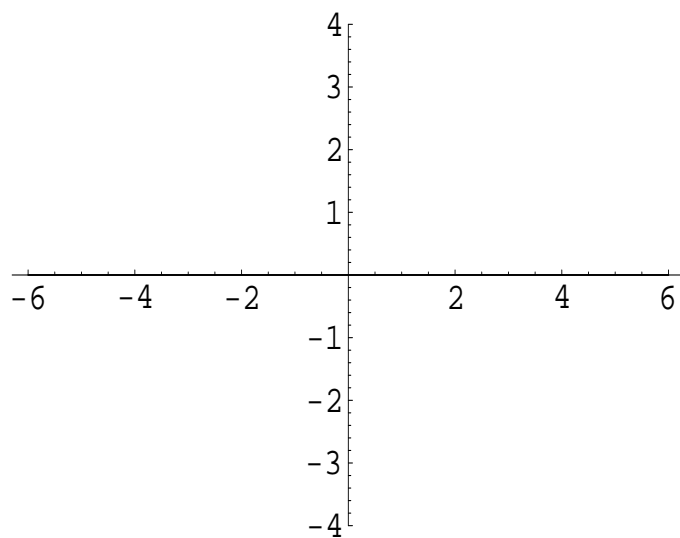
(e) $\lim_{x \rightarrow 2} g(x)$

(f) $\lim_{x \rightarrow -\infty} g(x)$

(g) List any vertical and horizontal asymptotes.

3. Sketch a graph of a function which satisfies **all** of the properties listed:

$$\lim_{x \rightarrow 0} f(x) = -2, f(0) = 1, \lim_{x \rightarrow 2^+} f(x) = -\infty, \lim_{x \rightarrow 2^-} f(x) = \infty, \lim_{x \rightarrow \pm\infty} f(x) = 0.$$



4. Solve the following equation for x .

$$\ln e^2 + e^{x^2} = e^{\ln 3}.$$

5. Find the value that $\lim_{x \rightarrow 3} g(x)$ must have if

$$(a) \lim_{x \rightarrow 3} \frac{g(x) + 2}{x - 3} = 2.$$

$$(b) \lim_{x \rightarrow 3} \frac{g(x) - 5}{x - 2} = 7.$$

6. Find the following limits.

$$(a) \lim_{x \rightarrow 0} \frac{3x}{\sin 5x}$$

$$(b) \lim_{x \rightarrow 0} \frac{\cos 3x}{x \csc 7x}$$

7. Determine if the following function has any points of discontinuity. For any such points, label the discontinuities as removable or non-removable.

$$f(x) = \frac{x^2 - x - 20}{x^2 - 16}$$

8. Let $f(x)$ be the function graphed below. Use this graph to find the following.

(a) $\lim_{x \rightarrow -3^+} f(x)$

(b) $\lim_{x \rightarrow 1^-} f(x)$

(c) $\lim_{x \rightarrow 1} f(x)$

(d) $\lim_{x \rightarrow 2^-} f(x)$

(e) $\lim_{x \rightarrow 2^+} f(x)$

(f) $\lim_{x \rightarrow 2} f(x)$

(f) $f(-3)$

(g) $f(1)$

9. For each of the following, sketch the graph of a function $f(x)$ that meets the stated criteria. If a function $f(x)$ cannot exist, say so.

(a) $\lim_{x \rightarrow 1^-} f(x) = -2$ but $\lim_{x \rightarrow 1^+} f(x) = 3$.

(b) $\lim_{x \rightarrow 2} f(x)$ does not exist, and $f(x)$ is continuous from the left at $x = 2$ but not from the right.

(c) $\lim_{x \rightarrow 2} f(x)$ exists, but $f(x)$ is not continuous at $x = 2$.

(d) $f(x)$ is differentiable at $x = 3$ but is not continuous at $x = 3$.

(e) $f'(x) = 0$ for all real numbers x .

10. Let $f(x) = x^2 + 2x + 3$.

(a) Determine $f'(x)$ using the **limit definition** of derivative.

(b) Find the slope of tangent line to $f(x)$ at $x = 1$. (You can use part (a).)

(c) Find the equation for the line tangent to $f(x)$ at $x = 1$.

11. Determine the **exact values** of each of the following.

(a) $\sin \frac{\pi}{3}$

(b) $\sin \frac{3\pi}{4}$

(c) $\cos \frac{3\pi}{2}$

(d) $\cos \frac{7\pi}{6}$

12. For what value of b is

$$g(x) = \begin{cases} bx, & x < 2 \\ x^2 + 1, & x \geq 2 \end{cases}$$

continuous at $x = 2$?