

Exam 2

Name: _____

Math 125.01
March 25, 2008

Question	Points Earned	Points Possible
1		20
2		15
3		5
4		6
5		6
6		8
7		10
8		7
9		7
10		12
11		6
Total		100

1. Find $\frac{dy}{dx}$ for the following functions.

(a) $y = e^{2x} \sin(3x)$

(b) $y = \frac{3x + 1}{x^2 + 1}$

(c) $y = \left(1 + \frac{1}{x}\right)^3$

(d) $y = \tan(\cos 2x)$

2. Find $g'(t)$ for the following functions.

(a) $g(t) = \ln(t^3 + 3 \sin t)$

(b) $g(t) = t \ln t - t$

(c) $g(t) = \ln\left(\frac{t+1}{t^3+1}\right)$

3. Find $f'(x)$ if $f(x) = \sec(e^{2x^3})$.

4. Calculate $f''(x)$ if $f(x) = \sin^2 x$.

5. Prove that if $f(x) = \tan x$, then $f'(x) = \sec^2 x$.
(You may use known derivatives of $\sin x$ and $\cos x$.)

6. Find the derivatives of each of the following. (Recall $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ and $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$.)

(a) $y = \sin^{-1} \sqrt{x}$

(b) $y = x^2 \tan^{-1} \sqrt{x-1}$

7. At the end of a basketball game, Lebron James throws the basketball directly upward in the air, and the ball's height is $h(t) = -8t^2 + 24t + 128$ feet after t seconds.

(a) What are the ball's velocity, speed, and acceleration after t seconds?

(b) How long does it take for the ball to reach its maximum height?

(c) What is its maximum height?

(d) How long does it take before the ball hits the ground?

8. Consider the function $f(x) = \sqrt{1+x}$.

(a) Find the linearization of f at $x = 3$.

(b) Use the linearization from part (a) to approximate $\sqrt{4.02}$.

9. A (21-year old) student is using a straw to drink beer out of a conical paper cup at a rate of 2 cubic centimeters per second. If the height of the cup is 12 centimeters and the diameter at the top of the cup is 8 centimeters, how fast is the beer level falling when the depth of the beer is 5 centimeters? (Recall: $V = \frac{1}{3}\pi r^2 h$ for a cone.)

10. Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = 2$ and $x = -3$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	-3	-5	$-1/2$	3
-3	1	7	4	-8

Find the derivatives below with respect to x at the given values. (Simplified answers are expected.)

(a) $h'(-3)$, where $h(x) = f(x) \cdot g(x)$

(b) $h'(2)$, where $h(x) = 7f(x) - 3g(x)$

(c) $h'(2)$, where $h(x) = g(f(x))$

(d) $h'(-3)$, where $h(x) = \ln(2x + g(x))$

11. Find the equation of the tangent line to the curve $x^3 + y^3 - 9xy = 0$ at the point $(2, 4)$.