Section 5.1 — 3, 7, 11, 15, 17, 19 — 2, 16

12 Points

2) $f(x) = x^3$, between 0 & 1

a) lower sum w/ 2 rectangles of equal width.

$\frac{1}{2}\left(0^3 + \left(\frac{1}{2}\right)^3\right) = \frac{1}{16} = .0625$ (1)

b) lower sum w/ 4 rectangles of equal width

$\frac{1}{4}\left(0^3 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3\right) = \frac{1}{4}\left(\frac{1}{64} + \frac{8}{64} + \frac{27}{64}\right)$

$= \frac{36}{256} = \frac{9}{64} = .140625$ (1)

c) upper sum w/ 2 rectangles

$\frac{1}{2}\left(\left(\frac{1}{2}\right)^3 + (1)^3\right) = \frac{1}{2}\left(\frac{1}{8} + 1\right)$

$= \frac{1}{2}\left(\frac{9}{8}\right)$ (1)

$= \frac{9}{16} = .5625$

d) upper sum w/ 4 rectangles

$\frac{1}{4}\left(\left(\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3 + (1)^3\right) = \frac{25}{64}$ (1)

$= .390625$

16) Average value of $f(x) = \frac{1}{x}$ on $[1, 9]$

Estimate with 4 subintervals of equal width and midpoints.

$\text{Sum} = 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)$

$= 2\left(\frac{25}{24}\right)$ (1)

$= \frac{25}{12} \approx 2.08\overline{3}$

$\text{Ave}_x \text{ Value} = \frac{\frac{25}{12}}{8} = \frac{25}{96}$ (1)

$\approx .2604$

Section 5.3: 1, 3, 9, 10, 11, 13, 15, 18, 23, 25 (10 problems)

10 Points

10) Suppose $\int_1^9 f(x)\,dx = -1$, $\int_7^9 f(x)\,dx = 5$, $\int_7^9 h(x)\,dx = 4$

a) $\int_1^9 -2f(x)\,dx = 2$

b) $\int_7^9 [f(x) + h(x)]\,dx = 5 + 4 = 9$

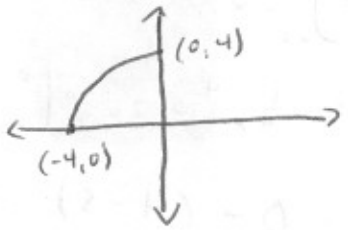
c) $\int_7^9 [2f(x) - 3h(x)]\,dx = 2(5) - 3(4) = -2$ (1/2 Point Each)

d) $\int_9^1 f(x)\,dx = 1$

e) $\int_1^7 f(x)\,dx = -1 - 5 = -6$

f) $\int_9^7 [h(x) - f(x)]\,dx = -4 - (-5)$
$= 1$

18) Use areas to determine $\int_{-4}^{0} \sqrt{16 - x^2}\,dx$



(2)

Area $= \frac{1}{4}\pi r^2$
$= \frac{1}{4}\pi (4)^2$
$= 4\pi$

Section 5.4: 1, 3, 5, 7, 9, 10, 17, 19, 21, 25, 27, 35, 37, 39, 41, 43, 44, 45, 47, 49, 51, 53, 54

(23 Problems)

14 Points

10) $\int_0^{\pi} (1+\cos x)\,dx = x + \sin x \Big|_0^{\pi}$ (2)

$= \pi + 0 - (0+0)$

$= \pi$

44) $y = \int_0^{x^2} \cos\sqrt{t}\,dt$

$\Rightarrow \frac{dy}{dx} = \left(\cos\sqrt{x^2}\right)(2x)$ (2)

$= 2x \cos|x|$

54) Find the total area between $y = x^3 - 4x$ and the x-axis where $-2 \le x \le 2$

$= x(x-2)(x+2)$

(1) For splitting

$\text{Area} = \int_{-2}^{0} x^3 - 4x\,dx - \int_0^{2} x^3 - 4x\,dx$ (1)

$= \frac{1}{4}x^4 - 2x^2 \Big|_{-2}^{0} - \frac{1}{4}x^4 - 2x^2 \Big|_0^{2}$

$= 0 - (4-8) - (4-8-0)$ (1)

$= 4 + 4$

$= 8$