

1. Find the following limits.

(a)  $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$

(b)  $\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1}$

(c)  $\lim_{x \rightarrow 0} \frac{\sin x}{x^3}$

(d)  $\lim_{x \rightarrow a} \frac{\sqrt[3]{x} - \sqrt[3]{a}}{x - a}, a \neq 0$

(e)  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - (x^2/2)}{x^3}$

(f)  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\tan(x^2)}$

(g)  $\lim_{x \rightarrow 0} \frac{\sin^{10} x}{\sin(x^{10})}$

(h)  $\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$

2. Prove that

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^n} = \infty$$

for any integer  $n$ . This shows that the exponential function approaches infinity faster than any power of  $x$ .

3. Prove that

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^p} = 0$$

for any number  $p$ . This shows that the logarithmic function approaches infinity more slowly than any power of  $x$ .

4. Find  $f(x)$ .

(a)  $f'(x) = \frac{2}{\sqrt{x^5}}$

(b)  $f''(x) = 1 + 2 \sin(x) - \cos(x), f(0) = 3, f'(0) = 1$

5. Find the function whose tangent line has slope  $3x^2 + 6x - 2$  for each value of  $x$  and whose graph passes through the point  $(0, 6)$ .

6. Find a function whose graph has a relative minimum when  $x = 1$  and relative maximum when  $x = 4$ .

7. Find the antiderivatives of the following functions in the most general form.

(a)  $f(t) = 6t^2\sqrt[3]{t}$

(b)  $h(x) = ax^2 + bx + c$

(c)  $k(x) = \frac{x^2 + 4x - 4}{\sqrt{x}}$

(d)  $g(x) = \frac{\sin x}{\cos^2 x}$

(e)  $h(t) = 3 \csc^2 t - 5 \sec t \tan t$

8. Write each of the following in sigma notation.

(a)  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

(b)  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$

(c)  $15 + 24 + 35 + \dots + (n^2 - 1)$

(d)  $1 + 3 + 5 + 7 + \dots + 21$

(e)  $2 + 4 + 8 + 16 + \dots + 1024$

(f)  $1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots + \frac{1}{x^n}$

(g)  $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$

(h)  $6 + 24 + 96 + 384 + \dots$

9. Express the following sums in sigma notation with  $i = 0$  as the lower limit of summation.

(a)  $\sum_{i=1}^{21} (i)^4$

(b)  $\sum_{i=2}^{17} \sin(\pi(i - 2))$

(c)  $\sum_{i=1000}^{1005} \frac{i}{100}$

(d)  $\sum_{i=-3}^{\infty} \frac{\sin(i)}{2^i}$

(e)  $\sum_{i=-2}^{18} (i^2 - 2i)$