Lab 10

1a) $$\lim_{x\to 2}\frac{x-2}{x^2-4} \overset{0/0}{=} \lim_{x\to 2}\frac{1}{2x} \qquad (L'H)$$

$$= \frac{1}{4}$$

b) $$\lim_{x\to 1}\frac{x^a-1}{x^b-1} \overset{0/0}{=} \lim_{x\to 1}\frac{ax^{a-1}}{bx^{b-1}} \qquad (L'H)$$

$$= \frac{a(1)^{a-1}}{b(1)^{b-1}} = \frac{a}{b}$$

c) $$\lim_{x\to 0}\frac{\sin x}{x^3} \overset{0/0}{=} \lim_{x\to 0}\frac{\cos x}{3x^2}^{\frac{1}{0}} \qquad (L'H)$$

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d) $$\lim_{x\to a}\frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a} \overset{0/0}{=} \lim_{x\to a}\frac{\frac{1}{3}x^{-2/3}}{1} \qquad (L'H)$$

$$= \frac{1}{3}a^{-2/3} = \frac{1}{3\cdot\sqrt[3]{a^2}}$$

e) $$\lim_{x\to 0}\frac{e^x-1-x-\frac{x^2}{2}}{x^3} \overset{0/0}{=} \lim_{x\to 0}\frac{e^x-1-x}{3x^2}^{0/0} \qquad (L'H)$$

$$= \lim_{x\to 0}\frac{e^x-1}{6x}^{0/0} \qquad (L'H)$$

$$= \lim_{x\to 0}\frac{e^x}{6} \qquad (L'H)$$

$$= \frac{e^0}{6} = \frac{1}{6}$$

f) $$\lim_{x\to 0}\frac{\sin^2 x}{\tan(x^2)} \overset{0/0}{=} \lim_{x\to 0}\frac{2\sin x\cos x}{2x\cdot\sec^2 x^2}^{0/0} \qquad (L'H)$$

$$= \lim_{x\to 0}\frac{\sin 2x}{2x\sec^2 x^2}^{0/0} \qquad (\text{Double-Angle Formula})$$

$$= \lim_{x\to 0}\frac{2\cos 2x}{2\sec^2 x^2 + 4x\sec x^2(\sec x^2\tan x^2)(2x)}$$

$$= \frac{2(1)}{2(1)+0} = 1$$

g) $\lim_{x\to 0} \frac{\sin^{10} x}{\sin(x^{10})} \overset{0/0}{=} \lim_{x\to 0} \frac{10\sin^9 x\cdot\cos x}{10x^9\cos(x^{10})}^{0/0}$ (L'H)

$= \lim_{x\to 0} \frac{90\sin^8 x\cos^2 x - 10\sin^{10} x}{90x^8\cos(x^{10}) - 100x^{18}\sin x^{10}}$ (L'H)

$= \lim_{x\to 0} \frac{\cancel{10}\sin^8 x\,(9\cos^2 x - \sin^2 x)}{\cancel{10}\,x^8\,(9\cos(x^{10}) - 10x^{10}\sin x^{10})}$

$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^8 \cdot \frac{(9\cos^2 x - \sin^2 x)}{(9\cos(x^{10}) - 10x^{10}\sin x^{10})}$

$= (1)\cdot\frac{9-0}{9-0} = 1$

h) $\lim_{x\to 0} \frac{\sin(mx)}{\sin(nx)} \overset{0/0}{=} \lim_{x\to 0} \frac{m\cos(mx)}{n\cos(nx)}$ (L'H)

$= \frac{m}{n}$

2) $\lim_{x\to\infty} \frac{e^x}{x^n} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{e^x}{nx^{n-1}} \overset{\infty/\infty}{}$ (L'H)

$= \lim_{x\to\infty} \frac{e^x}{n(n-1)x^{n-2}} \overset{\infty/\infty}{}$ (L'H)

$= \cdots = \lim_{x\to\infty} \frac{e^x}{n(n-1)(n-2)\cdots(1)x^0} = \infty$

3) $\lim_{x\to\infty} \frac{\ln x}{x^p} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{px^{p-1}}$ (L'H)

$= \lim_{x\to\infty} \frac{1}{px^p} = 0$

4) a) $f'(x) = \frac{2}{\sqrt{x^5}} = 2x^{-5/2} \Rightarrow f(x) = 2\cdot\frac{-2}{3}x^{-3/2} + C$

$= \frac{-4}{3}x^{-3/2} + C$

b) $f''(x) = 1 + 2\sin x - \cos x,\ f(0) = 3,\ f'(0) = 1$

$\Rightarrow f'(x) = x - 2\cos x - \sin x + C$

$f'(0) = 1 \Rightarrow 1 = 0 - 2 + C \Rightarrow C = 3 \Rightarrow f'(x) = x - 2\cos x - \sin x + 3$

$f(x) = \frac{1}{2}x^2 - 2\sin x + \cos x + 3x + D$

$f(0) = 3 \Rightarrow 3 = 0 - 0 + 1 + 0 + D \Rightarrow D = 2 \Rightarrow f(x) = \frac{1}{2}x^2 - 2\sin x + \cos x + 3x + 2$

5) $f'(x) = 3x^2 + 6x - 2$ , Passes through $(0,6) \Rightarrow f(0) = 6$

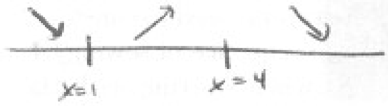
$\Rightarrow f(x) = x^3 + 3x^2 - 2x + C$

$\Rightarrow 6 = 0 + C$

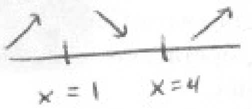
$\Rightarrow f(x) = x^3 + 3x^2 - 2x + 6$

6) $f$ must have a relative min @ $x=1$ and relative max @ $x=4$

we want this



If $f'(x) = (x-1)(x-4)$, we get



So $f'(x) = -(x-1)(x-4)$ will work.

$= -(x^2 - 5x + 4)$

$= -x^2 + 5x - 4$

$\Rightarrow f(x) = -\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x + C$

7 a) $\int 6t^2 \cdot \sqrt[3]{t}\, dt = \int 6t^{7/3} dt$

$= 6 \cdot \frac{3}{10} t^{10/3} + C = \frac{9}{5} t^{10/3} + C$

b) $\int ax^2 + bx + c\, dx = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx + D$

c) $\int \frac{x^2 + 4x - 4}{\sqrt{x}} dx = \int \frac{x^2}{x^{1/2}} + \frac{4x}{x^{1/2}} - \frac{4}{x^{1/2}} dx$

$= \int x^{3/2} + 4x^{1/2} - 4x^{-1/2} dx$

$= \frac{2}{5}x^{5/2} + \frac{8}{3}x^{3/2} - 8x^{1/2} + C$

d) $\int \frac{\sin x}{\cos^2 x} dx = \int \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} dx = \int \sec x \cdot \tan x\, dx$

$= \sec x + C$

e) $\int 3\csc^2 t - 5\sec t \cdot \tan t = -3\cot t - 5\sec t + C$

8 a) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \sum_{k=1}^{5}\frac{1}{k}$

b) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5} = \sum_{k=1}^{5}\frac{(-1)^{k+1}}{k}$

c) $15+24+35+\cdots+(n^2-1) = \sum_{j=4}^{n}(j^2-1)$

d) $1+3+5+7+\cdots+21 = \sum_{i=0}^{10}(2i+1) = \sum_{j=1}^{11}(2j-1)$

e) $2+4+8+16+\cdots+1024 = \sum_{k=1}^{10}2^k$

f) $1+\frac{1}{x}+\frac{1}{x^2}+\cdots+\frac{1}{x^n} = \sum_{k=0}^{n}\frac{1}{x^k}$

g) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots = \sum_{j=1}^{\infty}\frac{1}{3^j}$

h) $6+24+96+384+\cdots = \sum_{k=0}^{\infty}3\cdot 2^{2k+1}$

9 a) $\sum_{i=1}^{21}(i)^4$

$j=i-1 \Rightarrow i=j+1$

$i=1 \Rightarrow j=0$ $\quad i=21 \Rightarrow j=20$

$= \sum_{i=0}^{20}(i+1)^4$

b) $\sum_{i=2}^{17}\sin(\pi(i-2)) = \sum_{i=0}^{15}\sin(i\pi)$ $\quad j=i-2 \Rightarrow i=j+2$

c) $\sum_{i=1000}^{1005}\frac{i}{100} = \sum_{i=0}^{5}\frac{i+1000}{100}$ $\quad j=i-1000 \Rightarrow i=j+1000$

d) $\sum_{i=-3}^{\infty}\frac{\sin(i)}{2^i} = \sum_{i=0}^{\infty}\frac{\sin(i-3)}{2^{i-3}}$ $\quad j=i+3 \Rightarrow i=j-3$

e) $\sum_{i=-2}^{18}(i^2-2i) = \sum_{i=0}^{20}\left((i-2)^2-2(i-2)\right)$ $\quad j=i+2 \Rightarrow i=j-2$

$= \sum_{i=0}^{20}(i^2-4i+4-2i+4)$

$= \sum_{i=0}^{20}(i^2-6i+8)$