Lab 2

1a) $3 \ln(x^3 y) + 2 \ln \frac{y}{z^2} = \ln (x^3 y)^3 \cdot \left(\frac{y}{z^2}\right)^2 = \ln \frac{x^9 y^5}{z^4}$

b) $\frac{1}{2} \ln(x+3) - \ln x = \ln \frac{\sqrt{x+3}}{x} = 0 \Rightarrow \frac{\sqrt{x+3}}{x} = e^0 = 1 \Rightarrow \sqrt{x+3} = x$

Domain: $(0, \infty)$

$\Rightarrow x + 3 = x^2 \Rightarrow x^2 - x - 3 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+12}}{2}$

Since $x > 0$, it must be $x = \frac{1+\sqrt{13}}{2}$

c) $2 \cdot 5^{x/4} = 240 \Rightarrow 5^{x/4} = 120 \Rightarrow \frac{x}{4} \ln 5 = \ln 120 \Rightarrow x = \frac{4 \ln 120}{\ln 5}$

2 a) iii

b) ii

c) iv

d) i

3 a) $\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

b) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

c) $\arccos\left(\sin\left(\frac{\pi}{3}\right)\right) = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

d) $\tan\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = \tan\left(\frac{\pi}{4}\right) = 1$

4) $\cos t = \frac{-4}{5}, \quad \frac{\pi}{2} < t < \pi \Rightarrow \sin t = \sqrt{1 - \left(\frac{-4}{5}\right)^2} = \sqrt{\frac{25}{25} - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

a) $\cos 2t = \cos^2 t - \sin^2 t$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

Double-angle formulas

b) $\sin 2t = 2 \sin t \cos t$

$= 2\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right)$

$= \frac{-24}{25}$

c) $\sin\left(\frac{t}{2}\right) = +\sqrt{\frac{1-\cos t}{2}}$

$\frac{\pi}{2} < t < \pi \Rightarrow \frac{\pi}{4} < \frac{t}{2} < \frac{\pi}{2} \Rightarrow \sin\left(\frac{t}{2}\right) > 0$

Half-angle formula

$= \sqrt{\frac{1 - \frac{-4}{5}}{2}}$

$= \sqrt{\frac{\frac{9}{5}}{2}}$

$= \frac{3}{\sqrt{10}}$

	Average rate of change from	Let $h = 1$.	Let $h = .1$.	Let $h = .01$.	Let $h = .001$.	Let $h = .0001$.	What value is the average rate of change approaching?
Let $f(x) = x^2$.	3 to 3 + h	7	6.1	6.01	6.001	6.0001	6
$\frac{(4+h)^2 - 4^2}{h}$	4 to 4 + h	9	8.1	8.01	8.001	8.0001	8
$\frac{(x+h)^2 - x^2}{h}$	x to $x + h$	$2x+1$	$2x+.1$	$2x+.01$	$2x+.001$	$2x+.0001$	$2x$
Let $f(x) = \sqrt{x}$.	3 to 3 + h	$\frac{1}{2+\sqrt{3}}$	$\frac{1}{\sqrt{3.1}+\sqrt{3}}$	$\frac{1}{\sqrt{3.01}+\sqrt{3}}$	$\frac{1}{\sqrt{3.001}+\sqrt{3}}$	$\frac{1}{\sqrt{3.0001}+\sqrt{3}}$	$\frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$
$\frac{1}{\sqrt{4+h}+\sqrt{4}} = \frac{\sqrt{4+h} - \sqrt{4}}{h}$	4 to 4 + h	$\frac{1}{\sqrt{5}+2}$	$\frac{1}{\sqrt{4.1}+2}$	$\frac{1}{\sqrt{4.01}+2}$	$\frac{1}{\sqrt{4.001}+2}$	$\frac{1}{\sqrt{4.0001}+2}$	$\frac{1}{\sqrt{4}+2} = \frac{1}{4}$
$\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$	x to $x + h$	$\frac{1}{\sqrt{x+1}+\sqrt{x}}$	$\frac{1}{\sqrt{x+.1}+\sqrt{x}}$	$\frac{1}{\sqrt{x+.01}+\sqrt{x}}$	$\frac{1}{\sqrt{x+.001}+\sqrt{x}}$	$\frac{1}{\sqrt{x+.0001}+\sqrt{x}}$	$\frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$
Let $f(x) = \frac{1}{x}$.	3 to 3 + h	$\frac{-1}{3(4)} = \frac{-1}{12}$	$\frac{-1}{3(3.1)} = \frac{-1}{9.3}$	$\frac{-1}{3(3.01)} = \frac{-1}{9.03}$	$\frac{-1}{3(3.001)} = \frac{-1}{9.003}$	$\frac{-1}{3(3.0001)} = \frac{-1}{9.0003}$	$-\frac{1}{9}$
$\frac{-1}{4(4+h)} = \frac{\frac{1}{4+h} - \frac{1}{4}}{h}$	4 to 4 + h	$\frac{-1}{4(5)} = \frac{-1}{20}$	$\frac{-1}{4(4.1)} = -\frac{1}{16.4}$	$\frac{-1}{4(4.01)} = \frac{-1}{16.04}$	$\frac{-1}{4(4.001)} = \frac{-1}{16.004}$	$\frac{-1}{4(4.0001)} = \frac{-1}{16.0004}$	$-\frac{1}{16}$
$\frac{-1}{x(x+h)} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$	x to $x + h$	$\frac{-1}{x(x+1)}$	$\frac{-1}{x(x+.1)}$	$\frac{-1}{x(x+.01)}$	$\frac{-1}{x(x+.001)}$	$\frac{-1}{x(x+.0001)}$	$\frac{-1}{x^2}$