Math 125.01 Name:________________________________

Lab 3 - Sections 2.1, 2.2, 2.4

January 28, 2009

1. (From *Die Hard with a Vengeance*) You have a three gallon jug, a five gallon jug, and a unlimited supply of water. How can you fill the five gallon jug with exactly four gallons.

2. Compute the following limits (think about the *theorems* you use).

   $(a)\ \lim\limits_{x\to 1} \dfrac{x^2-1}{(x-1)^2}$ $(b)\ \lim\limits_{x\to -2} \dfrac{x^3+8}{x+2}$ $(c)\ \lim\limits_{x\to 1} \dfrac{x^3-2x^2+2x-1}{x^3-1}$

   $(d)\ \lim\limits_{x\to 2} \dfrac{x^2+4x+4}{x-2}$ $(e)\ \lim\limits_{x\to -1} \dfrac{\frac{1}{x}+1}{x+1}$ $(f)\ \lim\limits_{x\to 2} \dfrac{\sqrt{2}-\sqrt{x}}{2-x}$

   $(g)\ \lim\limits_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x^2}$

3. Find the following limits.

   (a) $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4}$

   (b) $\lim\limits_{x\to 0} \dfrac{\sin x}{x^3}$

   (c) $\lim\limits_{x\to \frac{3\pi}{2}^-} \dfrac{\cos x}{1-\sin x}$

   (d) $\lim\limits_{x\to 0} (\csc x - \cot x)$

   (e) $\lim\limits_{x\to 0} 6x^2(\cot x)(\csc 2x)$

   (f) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\sin 4x}$

   (g) $\lim\limits_{x\to 0} \dfrac{\sin mx}{\sin nx}$

4. For each part, sketch a graph of a function which satisfies **all** of the properties listed: (You should have one sketch for part (a) and one for part (b).)

   (a) $\lim\limits_{x\to 0} f(x) = 0,\ f(0) = 3,\ \lim\limits_{x\to 1^+} f(x) = -1,\ \lim\limits_{x\to 1^-} f(x) = 1,\ f(1) = 0.$

   (b) $\lim\limits_{x\to n^-} g(x) = n,\ \lim\limits_{x\to n^+} g(x) = n+1$, and $g(n) = n+1$ for every integer n.

5. Use the Squeeze Theorem to evaluate $\lim\limits_{x\to c} f(x)$ given that

$$\left|\frac{f(x)-f(c)}{x-c}\right| \leq M \text{ for } x \neq c.$$

6. Suppose that $g(x) \leq f(x) \leq h(x)$ for all $x \neq 2$ and suppose that

$$\lim_{x \to 2} g(x) = \lim_{x \to 2} h(x) = -5.$$

(a) Can we conclude anything about the values of f, g, and h at $x = 2$?

(b) Could $f(2) = 0$?

(c) Could $\lim_{x \to 2} f(x) = 0$?

7. We can easily observe that $\lim_{x \to 2}(x + 3) = 5$. Find the largest interval in which

(a) $|(x + 3) - 5| < 2$ (Hint: Think in terms of distance arguments.)

(b) $|(x + 3) - 5| < 1$

(c) $|(x + 3) - 5| < \varepsilon$ $(\varepsilon > 0)$

8. We can easily observe that $\lim_{x \to 4}(3x - 2) = 10$. Find the largest interval in which

(a) $|(3x - 2) - 10| < 2$

(b) $|(3x - 2) - 10| < 1$

(c) $|(3x - 2) - 10| < \varepsilon.$ $(\varepsilon > 0)$