Lab 3

2 a) $\lim_{x \to 1} \frac{x^2-1}{(x-1)^2} \overset{\frac{0}{0}}{=} \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{(x-1)^{\cancel{2}}} \quad \frac{2}{0}$ Does Not Exist

b) $\lim_{x \to -2} \frac{x^3+8}{x+2} \overset{\frac{0}{0}}{=} \lim_{x \to -2} \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}} = 12$ (Theorem 2, p. 81) + cancelling common factors

c) $\lim_{x \to 1} \frac{x^3-2x^2+2x-1}{x^3-1} \overset{\frac{0}{0}}{=} \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2-x+1)}{\cancel{(x-1)}(x^2+x+1)} = \frac{1}{3}$ (Theorem 3, p. 81)

d) $\lim_{x \to 2} \frac{x^2+4x+4}{x-2} \quad \frac{12}{0}$ Does Not Exist

e) $\lim_{x \to -1} \frac{\frac{1}{x}+1}{x+1} \overset{\frac{0}{0}}{=} \lim_{x \to -1} \frac{\frac{1+x}{x}}{x+1} = \lim_{x \to -1} \frac{\cancel{x+1}}{x\cancel{(x+1)}} = -1$ (Theorem 3, p. 81) + cancelling common factors

f) $\lim_{x \to 2} \frac{\sqrt{2}-\sqrt{x}}{2-x} \cdot \frac{\sqrt{2}+\sqrt{x}}{\sqrt{2}+\sqrt{x}} \overset{\frac{0}{0}}{=} \lim_{x \to 2} \frac{\cancel{2-x}}{\cancel{(2-x)}(\sqrt{2}+\sqrt{x})} = \frac{1}{2\sqrt{2}}$

g) $\lim_{x \to 0} \frac{1-\sqrt{1-x^2}}{x^2} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} \overset{\frac{0}{0}}{=} \lim_{x \to 0} \frac{\cancel{1}+(\cancel{-1}+x^2)}{x^2(1+\sqrt{1-x^2})}$

$= \lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}(1+\sqrt{1-x^2})} = \frac{1}{2}$

3 a) $\lim_{x \to 2} \frac{x-2}{x^2-4} \overset{\frac{0}{0}}{=} \lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \frac{1}{4}$

b) $\lim_{x \to 0} \frac{\sin x}{x^3}$ Does Not exist

c) $\lim_{x \to \frac{3\pi}{2}^-} \frac{\cos x}{1-\sin x} = \frac{0}{1--1} = \frac{0}{2} = 0$

d) $\lim_{x \to 0} \frac{\sin mx}{\sin nx} = \lim_{x \to 0} \frac{\sin mx}{mx} \cdot \frac{m}{n} \cdot \frac{nx}{\sin nx}$

$= \lim_{x \to 0} \frac{\sin mx}{mx} \cdot \lim_{x \to 0} \frac{m}{n} \cdot \lim_{x \to 0} \frac{nx}{\sin nx}$

$= 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n}$

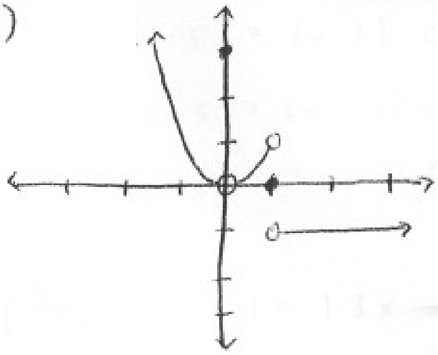
e) $\lim_{x \to 0} \csc x - \cot x = \lim_{x \to 0} \frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$= \lim_{x \to 0} \frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x \to 0} \frac{1-\cos^2 x}{\sin x(1+\cos x)}$

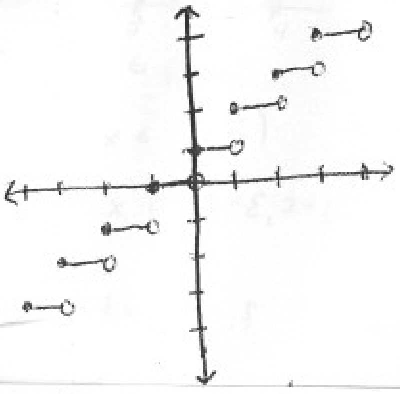
$= \lim_{x \to 0} \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}(1+\cos x)}$

$= \lim_{x \to 0} \frac{\sin x}{1+\cos x} = \frac{0}{1+1} = 0$

4a)



b)



5) $\left|\frac{f(x)-f(c)}{x-c}\right| \le M \Rightarrow -M \le \frac{f(x)-f(c)}{x-c} \le M$

$\Rightarrow -M(x-c) \le f(x)-f(c) \le M(x-c)$

$\Rightarrow -M(x-c)+f(c) \le f(x) \le M(x-c)+f(c)$

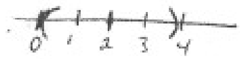
But $\lim_{x\to c} -M(x-c)+f(c) = \lim_{x\to c} M(x-c)+f(c) = f(c)$

Hence $\lim_{x\to c} f(x) = f(c)$ by the Squeeze Theorem.

6 a) No, although $\lim_{x\to 2} g(x) = \lim_{x\to 2} h(x) = -5$ and $g(x) \le f(x) \le h(x)$, we know nothing about what's going on at $x=2$ for any of the three functions

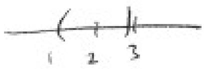
b) Yes (See part a)

c) No. by the Squeeze Theorem, $\lim_{x\to 2} f(x) = 0$

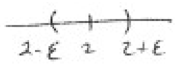


7 a) $|(x+3)-5| = |x-2| < 2$ if $x \in (0,4)$

b) $|(x+3)-5| = |x-2| < 1$ if $x \in (1,3)$

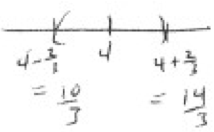


c) $|(x+3)-5| = |x-2| < \varepsilon$ if $x \in (2-\varepsilon, 2+\varepsilon)$



8 a) $|(3x-2)-10| = |3x-12| = 3|x-4| < 2$ if $|x-4| < \frac{2}{3}$

$x \in \left(\frac{10}{3}, \frac{14}{3}\right)$



b) $|(3x-2)-10| = 3|x-4| < 1$ if $x \in \left(\frac{11}{3}, \frac{13}{3}\right)$

c) $|(3x-2)-10| = 3|x-4| < \varepsilon$ if $x \in \left(4-\frac{\varepsilon}{3}, 4+\frac{\varepsilon}{3}\right)$