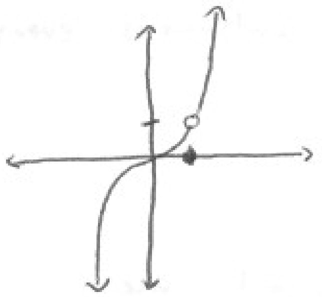
Lab 4

2 a)



b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 1$

c) Yes, it exists.

$\lim_{x\to 1} f(x) = 1$

3a) $\lim_{x\to 0} 6x^2 \cot x \cdot \csc 2x = \lim_{x\to 0} 6x^2 \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\sin 2x}$

$= \lim_{x\to 0} 6x^2 \cdot \frac{\cancel{\cos x}}{\sin x} \cdot \frac{1}{2\sin x \cancel{\cos x}}$

$= \lim_{x\to 0} 3 \cdot \frac{x}{\sin x} \cdot \frac{x}{\sin x} = 3(1)(1) = 3$

b) $\lim_{x\to 0} \frac{\sin 5x}{\sin 4x} = \lim_{x\to 0} \frac{\sin 5x}{5x} \cdot \frac{4x}{\sin 4x} \cdot \frac{5}{4} = (1)(1)(\frac{5}{4}) = \frac{5}{4}$

4a) $\lim_{x\to\pm\infty} \frac{2x^3+7}{x^3-x^2+x+7} = \lim_{x\to\pm\infty} \frac{\frac{2x^3}{x^3}+\frac{7}{x^3}}{\frac{x^3}{x^3}-\frac{x^2}{x^3}+\frac{x}{x^3}+\frac{7}{x^3}} = \lim_{x\to\pm\infty} \frac{2+\overbrace{\frac{7}{x^3}}^{\to 0}}{1-\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{7}{x^3}}_{\to 0}}$

$= \frac{2}{1} = 2$

b) $\lim_{x\to\pm\infty} \frac{3x+7}{x^2-2} = \lim_{x\to\pm\infty} \frac{\frac{3x}{x^2}+\frac{7}{x^2}}{\frac{x^2}{x^2}-\frac{2}{x^2}} = \lim_{x\to\pm\infty} \frac{\overbrace{\frac{3}{x}}^{\to 0}+\overbrace{\frac{7}{x^2}}^{\to 0}}{1-\underbrace{\frac{2}{x^2}}_{\to 0}} = \frac{0}{1} = 0$

5 a) $f(x) = \frac{3x-5}{2x^2-x-3} = \frac{3x-5}{(2x-3)(x+1)}$: Continuous except at $x = \frac{3}{2}, x = -1$

b) $f(x) = \frac{x^2-9}{x-3}$: Continuous except when $x = 3$

c) $f(x) = \frac{x}{x^2+1}$: Continuous for all real x ($x^2+1 \neq 0$ for all x)

d) $f(x) = \sqrt{2x-3} + x^2$: Continuous on its domain : $2x-3 \geq 0$ or $x \geq \frac{3}{2}$ or $[\frac{3}{2}, \infty)$

e) $f(x) = \frac{x}{\sqrt[3]{x-4}}$: Continuous everywhere except $x = 4$

6a) $f(x) = \frac{1}{x}$ and $g(x) = \frac{-1}{x}$ are discontinuous at $x=0$, but $(f+g)(x) = \frac{1}{x} + \frac{-1}{x} = 0$ is continuous everywhere (including $x=0$)

b) $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{2x}$, as well as $(f+g)(x) = \frac{1}{x} + \frac{1}{2x} = \frac{3}{2x}$ are discontinuous at $x=0$.

c) $f(x) = \begin{cases} -1 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$, $g(x) = \begin{cases} -2 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$, both discontinuous at $x=0$. $(fg)(x) = 2$ is continuous everywhere

d) $f(x) = g(x) = \frac{1}{x} \Rightarrow (fg)(x) = \frac{1}{x^2}$: All discontinuous at $x = 0$.

7a) $f(x)$ is continuous everywhere (and hence continuous from the left + right) since polynomial and rational functions are continuous everywhere, and

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(-1) = -1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 1.$$

b) $g(x)$ is not continuous from the left at $x=1$ since

$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} \frac{-1}{(x-1)^2} = -\infty \quad \text{but} \quad g(1) = 0$$ (Hence $g(x)$ is not continuous at $x=1$.)

$g(x)$ is, however, continuous from the right at $x=1$ since

$$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} \frac{x^2-1}{x+1} = 0 = g(1).$$

$g(x)$ is continuous everywhere except at $x=1$.

8 a) $f(x) = 5x-2$

$$f'(-1) = \lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0} \frac{5(-1+h)-2+7}{h}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}} = 5$$

$$f'(\tfrac{2}{3}) = \lim_{h\to 0} \frac{f(\frac{2}{3}+h)-f(\frac{2}{3})}{h} = \lim_{h\to 0} \frac{5(\frac{2}{3}+h)-2-\frac{4}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{\tfrac{10}{3}}+5h-\cancel{2}-\cancel{\tfrac{4}{3}}}{h} = 5$$

b) $g(x) = x^2$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

$$g'(1) = 2(1) = 2$$
$$g'(\tfrac{2}{3}) = 2(\tfrac{2}{3}) = \tfrac{4}{3}$$

9) $g(x) = \frac{1}{x} \Rightarrow g'(x) = \lim_{h\to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \dfrac{\frac{\cancel{x}}{x(x+h)} - \frac{\cancel{x}+h}{x(x+h)}}{h} = \lim_{h\to 0} \dfrac{\frac{-\cancel{h}}{x(x+h)}}{\cancel{h}}$

$$= \frac{-1}{x^2}$$

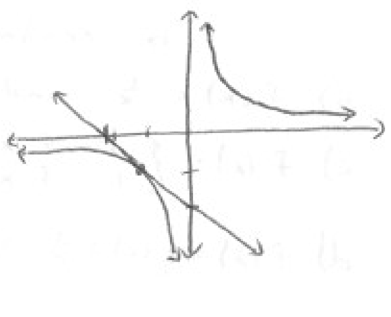
$$g'(-1) = \frac{-1}{(-1)^2} = -1$$

$g(-1)$

Tangent line: $y + 1 = -1(x - -1)$

$\Rightarrow y + 1 = -x - 1$

$\Rightarrow y = -x - 2$



Lab 4 (cont.)

10) $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

$= \lim_{h\to 0} \dfrac{(a+h)^2 - 2(a+h) + 3 - (a^2 - 2a + 3)}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{a^2} + 2ah + h^2 - \cancel{2a} - 2h + \cancel{3} - \cancel{a^2} + \cancel{2a} - \cancel{3}}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{h}(2a + h - 2)}{\cancel{h}} = 2a - 2 = 0$

if $\boxed{a = 1}$.

11) $h(t) = 58t - 0.83t^2$

$v(t) = h'(t) = \lim_{h\to 0} \dfrac{58(t+h) - 0.83(t+h)^2 - (58t - 0.83t^2)}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{58t} + 58h - \cancel{0.83t^2} - 1.66th + 0.83h^2 - \cancel{58t} + \cancel{0.83t^2}}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{h}(58 - 1.66t + 0.83h)}{\cancel{h}} = 58 - 1.66t$

a) $v(1) = 58 - 1.66(1) = 56.34$

b) $v(a) = 58 - 1.66a$

c) $h(t) = 0$ if $58t - 0.83t^2 = t(58 - 0.83t) = 0 \Rightarrow t = 0$ or $t \approx 69.88$ sec

The arrow will hit the moon after about 69.88 sec.

d) $v(69.88) = 58 - 1.66(69.88) \approx -58.00$ m/s