Lab 5

2) Units on $B = \$$; Units on t = years

$\Rightarrow$ Units on $\frac{dB}{dt} = \frac{\text{dollars}}{\text{yr}}$

$\frac{dB}{dt}$ = the rate at which your balance is changing with respect to time

3) $P = f(t) = 1.15(1.014)^t$

$f(6) = 1.15(1.014)^6 \approx 1.25$, which means that China's population was about 1.25 billion in 1999.

$P' = f'(t) = 1.15(1.014)^t \cdot \ln 1.014$

$f'(6) = 1.15(1.014)^6 \cdot \ln 1.014 \approx .01738$, which means that in 1999, China's population was increasing at a rate of about 17.38 million (.01738 billion) people per year.

4) $s(t) = 6\sin(2t) = 12\sin t \cos t$ (Double-angle formula)

a) $v(t) = s'(t) = 12\cos^2 t - 12\sin^2 t$

b) $v(t) = 0 \Rightarrow 12\cos^2 t - 12\sin^2 t = 0 \Rightarrow \sin^2 t = \cos^2 t$

$\Rightarrow \sin t = \pm \cos t$

$\Rightarrow t = \frac{(2k+1)\pi}{4}$, where k is an integer

$\left(t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots\right)$

c) maximum distance = 6 units since $\sin(2t) \leq 1$ for all t.

5a) $Q(t) = 20(300 - t^2) = 6000 - 20t^2$

$\Rightarrow Q'(t) = -40t \Rightarrow Q'(10) = -400$ gal/min

b) Average rate $= \frac{Q(10) - Q(0)}{10 - 0}$

$= \frac{4000 - 6000}{10}$

$= \frac{-2000}{10} = -200$ gal/min

6) $D = \frac{10}{9}t^2 \Rightarrow D' = \frac{20}{9}t = \frac{500}{9} \Rightarrow t = 25$ sec

Distance travelled $= D(25) = \frac{10}{9}(25)^2 = \frac{6250}{9} \approx 694.4$ m

$200 \text{ km/hr} \cdot 1000 \text{ m/km} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{500}{9}$ m/s

7) $f'(x) = 1 + 2\cos x = 0 \Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow x = \frac{2\pi}{3} + 2k\pi$ or $x = \frac{4\pi}{3} + 2k\pi$

where k is an integer.

8 a) $y = \cos x \cdot \tan x$

$$\Rightarrow \frac{dy}{dx} = \cos x\,(\sec^2 x) + \tan x\,(-\sin x)$$

$$= \cancel{\cos x} \cdot \frac{1}{\cancel{\cos^2} x} + \frac{\sin x}{\cos x} \cdot -\sin x$$

$$= \frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x}$$

$$= \cos x$$

b) $y = \cos x \cdot \tan x$

$$= \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} = \sin x$$

$$\Rightarrow \frac{dy}{dx} = \cos x$$

9 a) $f'(x) = 24x^2 - 2x$

b) $g'(x) = 6a^2x + 3x^2$

c) $h'(x) = \pi + 2\pi^3 x$

d) $f(x) = \frac{1}{2}x^2 + \frac{5}{2} \Rightarrow f'(x) = x$

e) $g(x) = x^{1/2}(x^2-1) = x^{5/2} - x^{1/2} \Rightarrow g'(x) = \frac{5}{2}x^{3/2} - \frac{1}{2}x^{-1/2}$

f) $h'(x) = \dfrac{(x-2)(1) - (x+2)(1)}{(x-2)^2} = \dfrac{-4}{(x-2)^2}$

g) $f(x) = \dfrac{(8x+2)(x+1)}{x-3} = \dfrac{8x^2+10x+2}{x-3}$

$$\Rightarrow f'(x) = \frac{(x-3)(16x+10) - (8x^2+10x+2)(1)}{(x-3)^2}$$

$$= \frac{16x^2 + \cancel{10x} - 48x - 30 - 8x^2 - \cancel{10x} - 2}{(x-3)^2} = \frac{8x^2 - 48x - 32}{(x-3)^2} = \frac{8(x^2-6x-4}{(x-3)^2}$$

h) $g'(x) = \dfrac{(x^2-1)[(2)(3x+2) + (2x+1)(3)] - (2x+1)(3x+2)(2x)}{(x+1)^2(x-1)^2}$

9 i) $h(x) = x^2 + 1 - 2x - 2x^{-1} \Rightarrow h'(x) = 2x - 2 + 2x^{-2}$

j) $f'(x) = (5x^4 - 2x^{-3})(x^3 - x^{-7}) + (x^5 + x^{-2})(3x^2 + 7x^{-8})$

k) $g'(t) = \dfrac{(t^3+3t^2+3)(2t+1) - (t^2+t-4)(3t^2+6t)}{(t^3+3t^2+3)^2}$

l) $h(x) = \dfrac{x^{-3} - x^4}{x^4} = \dfrac{x^{-3}}{x^4} - \dfrac{x^4}{x^4} = x^{-7} - 1$

$\Rightarrow h'(x) = -7x^{-8}$

m) $f'(x) = 2x \cdot 3^x + x^2 \cdot 3^x \ln 3$

10 a) $\dfrac{dy}{dx} = -\sin x - 2\csc^2 x - 6x$

b) $\dfrac{dy}{dx} = \sin x + x\cos x$

c) $\dfrac{dy}{dx} = 2(\sqrt{x} - \cot x) + 2x\left(\frac{1}{2}x^{-1/2} + \csc^2 x\right)$

$= 2\sqrt{x} - 2\cot x + \sqrt{x} + 2x\csc^2 x = 3\sqrt{x} - 2\cot x + 2x\csc^2 x$

d) $y = x\cos^2 x = x(\cos x)(\cos x)$

$\dfrac{dy}{dx} = (1)\cos^2 x + x(-\sin x)(\cos x) + x(\cos x)(-\sin x)$

$= \cos^2 x - 2x\sin x \cdot \cos x$

e) $\dfrac{dy}{dx} = \dfrac{x^3(1+\cos x) - (x+1)[x^3(1+\cos x)]'}{x^6(1+\cos x)^2}$

$= \dfrac{x^3(1+\cos x) - (x+1)[3x^2(1+\cos x) + x^3(-\sin x)]}{x^6(1+\cos x)^2}$