Math 125.01 Name:_____
Lab 6 - Sections 3.4, 3.5, & 3.6
February 25, 2009

1. *(Gnomes and the Evil Rock King)* A gaggle of 10 happy gnomes happened into the domain of the evil Rock King one afternoon. The Rock King, being evil, ordered that all the gnomes be captured and brought before him. This was done and as the gnomes stood before him, he offered a challenge. "Tomorrow," said the Rock King, "I shall stand you in a single file line and place a hat upon your head which will be either black or white, the color unknown to you. Beginning with the last of you in the line, I shall ask you the color of the hat upon your head. Should you answer my question correctly, I will spare your life. Otherwise, you will die." The gnomes were then sent to the dungeon until the morning. In the dungeon, the gnomes encountered a troll who offered a scheme which would save the maximum number of gnomes. What is the scheme the troll offered, and how many gnomes will be guaranteed to have their lives spared?

2. Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, with $f(2) = 3$, $f(5) = -7$, $f'(2) = -2$, $f'(5) = 11$, $g(2) = 5$, $g(3) = -4$, $g'(2) = 2$, and $g'(3) = 5$. Find $h'(2)$ and $F'(2)$.

3. Does $\frac{d}{dx}\cos^2 x = \frac{d}{dx}\cos x^2$? Justify your answer.

4. Differentiate the following functions.

   (a) $f(x) = (x^2 + 4x - 5)^4$

   (b) $g(x) = 4\cos 3x - 3\sin 4x$

   (c) $h(x) = \cos(3x^2 + 1)$

   (d) $p(y) = (y + 3)^3(5y + 1)^2(3y^2 - 4)$

   (e) $f(t) = \left(\dfrac{2t^2 + 1}{3t^3 + 1}\right)^2$

   (f) $f(x) = \sin^2(\cos 3x)$

   (g) $z(x) = (1 + (1 + (1 + (1 + \sin x)^2)^3)^4)^5$

   (h) $y = x^2e^{-x}$

   (i) $f(x) = xe^x - e^x$

   (j) $g(x) = \dfrac{\sqrt{x}}{\sin 2x + e^{5x}}$

   (k) $h(x) = e^{3x}(\sin 5x + \cos 2x)$

5. Find an expression for $\dfrac{dy}{dx}$ for each of the following curves.

   (a) $4x^2 - 9y^2 = 36$

   (b) $x^{2/3} + y^{2/3} = 1$

   (c) $y = \cos(x - y)$

   (d) $x\sin y + y\cos x = 1$

6. Find the equations of both of the tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$. (Note: $(12, 3)$ is not a point on the ellipse.)