Lab 6

2) $h(x) = f(x)\,g(x) \Rightarrow h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

$h'(2) = f'(2)\,g(2) + f(2)\,g'(2) = (-2)(5) + (3)(2) = -4$

$F(x) = (f \circ g)(x) = f(g(x)) = F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'(g(2)) \cdot g'(2)$

$= f'(5) \cdot 2 = 11(2) = 22$

3) $\frac{d}{dx}(\cos^2 x) = 2\cos x\,(-\sin x) = -2\cos x \cdot \sin x$

$\frac{d}{dx}(\cos x^2) = -\sin x^2\,(2x) = -2x \sin x^2$ No; they're not equal.

4) a) $f(x) = (x^2+4x-5)^4 \Rightarrow f'(x) = 4(x^2+4x-5)^3(2x+4)$

b) $g(x) = 4\cos 3x - 3\sin 4x \Rightarrow g'(x) = -12\sin 3x - 12\cos 4x$

c) $h(x) = \cos(3x^2+1) \Rightarrow h'(x) = -6x\sin(3x^2+1)$

d) $p(y) = (y+3)^3(5y+1)^2(3y^2-4)$

$\Rightarrow p'(y) = 3(y+3)^2(1)(5y+1)^2(3y^2-4) + (y+3)^3 \cdot 2(5y+1)(5)(3y^2-4)$
$+ (y+3)^3(5y+1)^2(6y)$

$= (y+3)^2(5y+1)\left[3(5y+1)(3y^2-4) + 10(y+3)(3y^2-4) + 6y(y+3)(5y+1)\right]$

e) $f(t) = \left(\frac{2t^2+1}{3t^3+1}\right)^2 \Rightarrow f'(t) = 2\left(\frac{2t^2+1}{3t^3+1}\right)\left(\frac{(3t^3+1)(4t) - (2t^2+1)(9t^2)}{(3t^3+1)^2}\right)$

f) $f(x) = \sin^2(\cos 3x) \Rightarrow f'(x) = 2\sin(\cos 3x) \cdot \cos(\cos 3x) \cdot (-\sin 3x)(3)$

$= -6\sin(\cos 3x) \cdot \cos(\cos 3x) \cdot \sin 3x$

g) $z(x) = (1+(1+(1+(1+\sin x)^2)^3)^4)^5$

$\Rightarrow z'(x) = 5(1+(1+(1+(1+\sin x)^2)^3)^4)^4\,4(1+(1+(1+\sin x)^2)^3)^3\,3(1+(1+\sin x)^2)^2\,2(1+\sin x) \cdot co$

h) $y = x^2 e^{-x} \Rightarrow \dfrac{dy}{dx} = 2xe^{-x} - x^2 e^{-x}$ (Product Rule + Chain Rules)

i) $f(x) = xe^x - e^x \Rightarrow f'(x) = (1)e^x + xe^x - e^x$ (Product Rule + Chain Rules)

$= xe^x$

j) $g(x) = \dfrac{\sqrt{x}}{\sin 2x + e^{5x}} = \dfrac{x^{1/2}}{\sin 2x + e^{5x}} \Rightarrow g'(x) = \dfrac{(\sin 2x + e^{5x})\frac{1}{2}x^{-1/2} - x^{1/2}(2\cos 2x + 5e^{5x})}{(\sin 2x + e^{5x})}$

(Quotient + Chain Rules)

$= \dfrac{\dfrac{\sin 2x + e^{5x}}{2\sqrt{x}} - \sqrt{x}(2\cos 2x + 5e^{5x})}{(\sin 2x + e^{5x})}$

k) $h(x) = e^{3x}(\sin 5x + \cos 2x)$ (Product and Quotient Rules)

$\Rightarrow h'(x) = 3e^{3x}(\sin 5x + \cos 2x) + e^{3x}(5\cos 5x - 2\sin 2x)$

5 a) $4x^2 - 9y^2 = 36$

$$\Rightarrow 8x - 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{8x}{18y} = \frac{4x}{9y}$$

b) $x^{2/3} + y^{2/3} = 1$

$$\Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}} = \frac{-y^{1/3}}{x^{1/3}} = -\sqrt[3]{\frac{y}{x}}$$

c) $y = \cos(x-y)$

$$\Rightarrow \frac{dy}{dx} = -\sin(x-y)\cdot\left(1-\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = -\sin(x-y) + \sin(x-y)\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(1-\sin(x-y)\right) = -\sin(x-y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x-y)}{1-\sin(x-y)}$$

d) $x\sin y + y\cos x = 1$

$$\Rightarrow (1)\cdot\sin y + x\cos y\frac{dy}{dx} + \frac{dy}{dx}\cos x - y\sin x = 0$$

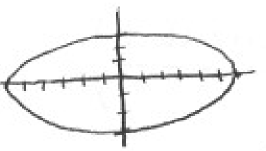
$$\Rightarrow \frac{dy}{dx}(x\cos y + \cos x) = y\sin x - \sin y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin x - \sin y}{x\cos y + \cos x}$$

6) $x^2+4y^2=36$

$$2x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2x}{8y} = -\frac{x}{4y}$$

(12, 3)



In Point-slope form, the equation of any tangent line to $x^2+4y^2=36$ passing through (12, 3) is

By inspection, $y=3$ is one such line

Also, both tangent lines are on the right side of the ellipse, so $x = \sqrt{36-4y^2}$

$$y-3 = \frac{-x}{4y}(x-12)$$

$$= \frac{-\sqrt{36-4y^2}}{4y}\left(\sqrt{36-4y^2}-12\right)$$

$$= \frac{-36+4y^2+12\sqrt{36-4y^2}}{4y}$$

$$\Rightarrow \cancel{4y^2} - 12y = -36 + \cancel{4y^2} + 12\sqrt{36-4y^2}$$

$$\Rightarrow y = 3 - \sqrt{36-4y^2} \Rightarrow (y-3)^2 = \left(-\sqrt{36-4y^2}\right)^2$$

$$\Rightarrow y^2 - 6y + 9 = 36 - 4y^2$$

$$\Rightarrow 5y^2 - 6y - 27 = 0 \Rightarrow (5y+9)(y-3) = 0 \Rightarrow y = \frac{-9}{5} \text{ or } y = 3$$

$\boxed{y=3}$ or

$$\frac{-24/5}{4(-\frac{9}{5})}$$

$$y = \frac{-9}{5} \Rightarrow x = \sqrt{36-4(\frac{-9}{5})^2} = \frac{24}{5} \Rightarrow \boxed{y + \frac{9}{5} = \frac{2}{3}\left(x - \frac{24}{5}\right)}$$