Math 125.02 Name:_____
Lab 7 - Sections 3.7-3.9
March 5, 2009

1. (From *The Heart of Mathematics*)
   On an archaeological dig near the highlands of Tibet, Alley discovered an ancient oil lamp. Just for laughs she rubbed the lamp. She quickly stopped laughing when a huge puff of magenta smoke spouted from the lamp, and an ornery genie named Murray appeared. Murray, looking at the stunned Alley, exclaimed, "Well, what are you staring at? Okay, okay, you've found me; you get your three wishes. So, what will they be?" Alley, although in shock, realized what an incredible opportunity she had. Thinking quickly, she said, "I'd like to find the Rama Nujan, the jewel that was first discovered by Hardy the High Lama." "You got it," replied Murray, and instantly nine identical-looking stones appeared. Alley looked at the stones and was unable to differentiate any one from the others.

   Finally she said to Murray, "So where is the Rama Nujan?" Murray explained, "It is embedded in one of these stones. You said you wished to find it. So now you have to find it. Oh, by the way, you may take only one of the stones with you, so you had best be careful how you choose!"

   "But they look identical to me. How will I know which one has the Rama Nujan in it?" Alley questioned. "Well, eight of the stones weigh the same, but the stone containing the jewel weighs slightly more than the others," Murray responded with a devilish grin.

   Alley, now getting annoyed, whispered under her breath, "Gee, I wish I had a balance scale." Suddenly a balance scale appeared. "That was wish two!" Realizing he had gone a bit overboard, Murray proclaimed, "Hey, I want to help you out, so let me give you a tip: That balance scale can only be used once." "What? Only once?" she said, thinking out loud. "I wish I had another balance scale." "Okay, kiddo, that was wish three." Murray snickered. "Just like the other one, this is a 'wished' balance scale. That means you can use it only once since it was one wish."

   So, Alley may use each of the two balance scales exactly once. Is it possible for Alley to select the slightly heavier stone containing the Rama Nujan from among the nine identical-looking stones? Please explain why or why not.

2. Find the derivatives of the following functions.

   (a) $f(x) = \tan^{-1}\sqrt{x}$

   (b) $h(t) = \tan(\sin^{-1} t)$

WARNING: Whenever solving problems asking you to determine the *rate of change* of a specific quantity, that rate must be measured using some kind of units. (i.e. miles per hour, cubic meters per second, etc.) Be sure to always state the units in any practical application problem.

3. A stone is dropped into a still pond. Concentric circular ripples spread out, and the radius of the disturbed region increases at the rate of 16 cm/sec. At what rate does the area of the disturbed region increase when its radius is 4 cm?

4. A horizontal trough is 16 m long, and its ends are isosceles trapezoids with an altitude of 4 m, a lower base of 4 m, and an upper base of 6 m. Water is being poured into the trough at the rate of 10 m^3/min. How fast is the water level rising when the water is 2 m deep?

5. Water is poured at the rate of 8 ft^3/min into a tank in the shape of a cone. The cone is 20 ft deep and 10 ft in diameter at the top. If there is a leak in the bottom of the tank and the water level is rising at the rate of 1 in/min, when the water is 16 ft deep, how fast is the water leaking?

6. Show that if the volume of a spherical balloon is decreasing at a rate proportional to its surface area, then the radius of the balloon is shrinking at a constant rate.

7. A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

   (a) How fast is the top of the ladder sliding down the wall then?

   (b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

   (c) At what rate is the angle θ between the ladder and the ground changing then?

8. A swimming pool is 15 ft wide, 40 ft long, 3 ft deep at one end, and 10 ft deep at the other. Draw a sketch of the pool. Water is being added to the pool at a rate of 25 ft^3/min.

   (a) How fast is the water level rising when there is 2000 ft^3 of water in the pool?

   (b) How fast is the water level rising when there is 3000 ft^3 of water in the pool?

9. Find the derivative of y with respect to x or t, as appropriate.

   (a) $y = \ln(kx)$ (k is constant)
   (b) $y = (\ln x)^3$
   (c) $y = t\sqrt{\ln t}$
   (d) $y = \ln(\ln(\ln x))$
   (e) $y = \sqrt{\dfrac{(x+1)^5}{(x+2)^{20}}}$