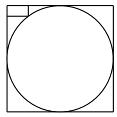
Math 125.02 Name:______________________________
Lab 8 - Sections 4.1-4.2
March 25, 2009

1. A circle is inscribed in a square as pictured below. The rectangle at the corner measures 1 cm $\times$ 2 cm. What is the radius of the circle in cm?



2. For each of the following, show that f satisfies the hypotheses of Rolle's Theorem on the given interval $[a, b]$, and find all values $c \in (a, b)$ for which $f'(c) = 0$:

   (a) $f(x) = 3x^2 - 12x + 11 \quad [0, 4]$

   (b) $f(\theta) = \sin(2\theta) \quad [0, \pi]$

3. Let $f(x) = (x - 1)^{-2}$. Show that $f(0) = f(2)$, but there is not number c in $(0, 2)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

4. For each of the following, show that g satisfies the hypotheses of the Mean Value Theorem on the given interval $[a, b]$ and find all of the values $c \in (a, b)$ for which $g(b) - g(a) = g'(c)(b - a)$.

   (a) $g(x) = \dfrac{1}{(x+1)^2} \quad [0, 2]$

   (b) $g(z) = 4 + \sqrt{z - 1} \quad [1, 5]$

5. A straight highway 50 miles long connects two cities, A and B. Show that it is impossible to travel from A to B by car in exactly one hour without having the speedometer register 50 mph at least once.

6. Does there exist a function f such that $f(0) = -1, f(2) = 4$, and $f'(x) \leq 2$ for all x?

7. Does a function always attain its maximum or minimum in a closed interval? Construct some examples and explain them.

8. Find the absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

   (a) $f(x) = \dfrac{2}{3}x - 5, \quad -2 \leq x \leq 3$

   (b) $f(x) = -\dfrac{1}{x^2}, \quad 0.5 \leq x \leq 2$

   (c) $g(x) = \csc x, \quad \dfrac{\pi}{3} \leq x \leq \dfrac{2\pi}{3}$