Lab 8

2a) $f(x) = 3x^2 - 12x + 11$

Since $f(x)$ is a polynomial function, it clearly is continuous + differentiable everywhere.

$f(0) = f(4) = 11$.

Hence Rolle's Theorem implies that there exists $c \in (0,4)$ such that $f'(c)$

$f'(x) = 6x - 12 \Rightarrow f'(c) = 0$ for some c

$\Rightarrow 6c - 12 = 0 \Rightarrow c = 2$

b) $f(\theta) = \sin(2\theta)$ is continuous + differentiable everywhere.

$f(0) = \sin(0) = \sin(2\pi) = 0$

Hence, Rolle's Theorem implies that there exists $c \in (0,\pi)$ such that $f'(c) = 0$.

$f'(\theta) = 2\cos(2\theta) \Rightarrow f'(c) = 2\cos(2c) = 0$ for some c

$\Rightarrow c = \frac{\pi}{4}$ or $c = \frac{3\pi}{4}$

3) If $f(x) = (x-1)^{-2} = \frac{1}{(x-1)^2}$, then $f(0) = 1 = f(2)$, but

$f'(x) = \frac{-2}{(x-1)^3} \neq 0$ for all x.

Rolle's Theorem is not violated since $f(x)$ is not continuous @ $x = 1$.

4a) $g(x) = \frac{1}{(x+1)^2}$ is continuous and differentiable everywhere except at $x = -1$.

Hence, MVT applies.

$g(0) = 1$, $g(2) = \frac{1}{9}$, and $g'(x) = \frac{-2}{(x+1)^3}$

We need c such that $g'(c) = \frac{-2}{(c+1)^3}$ and $\frac{1}{9} - 1 = \frac{-2}{(c+1)^3}(2-0)$

$\Rightarrow -\frac{8}{9} = \frac{-4}{(c+1)^3} \Rightarrow (c+1)^3 = \frac{-36}{-8} = \frac{9}{2}$

$\Rightarrow c+1 = \sqrt[3]{\frac{9}{2}} \Rightarrow c = \sqrt[3]{\frac{9}{2}} - 1$

b) $g(z) = 4 + \sqrt{z-1}$ is continuous on $[1,\infty)$ and differentiable on $(1,\infty)$.

Hence, MVT applies, and $g(1) = 4$, $g(5) = 6$, $g'(z) = \frac{1}{2}(z-1)^{-1/2} = \frac{1}{2\sqrt{z-1}}$

We need c so that $6 - 4 = \frac{1}{2\sqrt{c-1}}(5-1)$

$\Rightarrow 2 = \frac{4}{2\sqrt{c-1}} \Rightarrow \sqrt{c-1} = 1$

$\Rightarrow c - 1 = 1 \Rightarrow c = 2.$

5) Let $s(t)$ = distance from A (in miles) after t hours.

Then $s(0)=0$, $s(1)=50$, and $s(t)$ is continuous and differentiable for all t.

The MVT $\Rightarrow$ $s(1)-s(0)=s'(c)(1-0)$ for some $c \in (0,1)$.

$\Rightarrow 50-0=s'(c)(1)$

$\Rightarrow s'(c)=50$ for some $c \in (0,1)$.

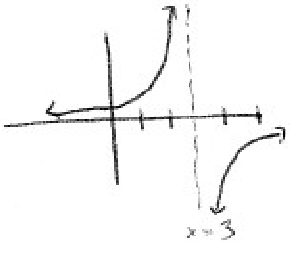
6) Since f satisfies the hypotheses of the MVT, ($f'(x) \le 2$ for all x $\Rightarrow$ f is continuous & differentiable everywhere),

we know that there is some point $c \in (0,2)$ such that $f'(c) = \frac{f(2)-f(0)}{2-0}$

$= \frac{4-{-1}}{2} = \frac{5}{2} > 2$

Hence, it cannot be the case that $f'(x) \le 2$ for all x.

7) This is only guaranteed if the function is continuous on the closed interval.



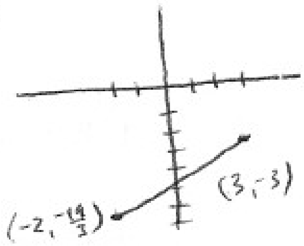
This function does not attain its minimum or maximum on $[1,5]$.

8 a) $f(x) = \frac{2}{3}x - 5$, $-2 \le x \le 3$

$f'(x) = \frac{2}{3} \neq 0$ for all x (and exists for all $x \in [-2, 3]$)

$f(-2) = \frac{-4}{3} - \frac{15}{3} = \frac{-19}{3}$ (Local and absolute min)

$f(3) = 2 - 5 = -3$ (Local and absolute max)

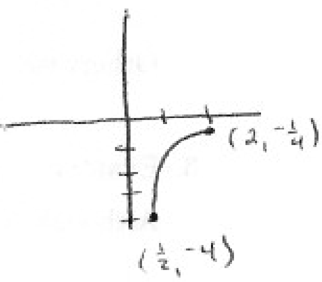


b) $f(x) = \frac{-1}{x^2} = -x^{-2}$, $\frac{1}{2} \le x \le 2$

$f'(x) = 2x^{-3} = \frac{2}{x^3} \neq 0$ for all x (and exists for all $x \in [\frac{1}{2}, 2]$

$f(\frac{1}{2}) = \frac{-1}{(\frac{1}{4})} = -4$ (Local + absolute min)

$f(2) = \frac{-1}{(2)^2} = \frac{-1}{4}$ (Local + absolute max)



c) $g(x) = \csc x$, $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$

$g'(x) = -\csc x \cdot \cot x = \frac{-\cos x}{\sin^2 x} = 0$ if $x = \frac{\pi}{2}$

(and exists for all $x \in [\frac{\pi}{3}, \frac{2\pi}{3}]$

$g(\frac{\pi}{2}) = \frac{1}{1} = 1$ (Local and absolute min)

$g(\frac{\pi}{3}) = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ (Local and absolute max)

$g(\frac{2\pi}{3}) = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ (Local and absolute max)