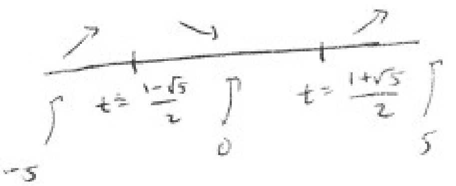
1 a) $f(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 - t - 1$

$\Rightarrow f'(t) = t^2 - t - 1 = 0 \Rightarrow t = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$



$f'(-5) = 29 > 0$

$f'(0) = -1 < 0$

$f'(5) = 19 > 0$

$f(\frac{1+\sqrt{5}}{2}) \approx -2.515$

$f(\frac{1-\sqrt{5}}{2}) \approx$

Increasing: $(-\infty, \frac{1-\sqrt{5}}{2}] \cup [\frac{1+\sqrt{5}}{2}, \infty)$

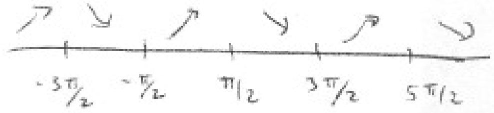
Decreasing: $[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}]$

Local min $(\frac{1+\sqrt{5}}{2}, -2.515)$

Local max $(\frac{1-\sqrt{5}}{2}, -.652)$

b) $g(x) = \sin x$

$\Rightarrow g'(x) = \cos x = 0$ if $x = \frac{k\pi}{2}$ where k is an odd integer



Increasing: $\cdots \cup [-\frac{5\pi}{2}, -\frac{3\pi}{2}] \cup [-\frac{\pi}{2}, \frac{\pi}{2}] \cup [\frac{3\pi}{2}, \frac{5\pi}{2}] \cup \cdots$

Decreasing: $\cdots \cup [-\frac{3\pi}{2}, -\frac{\pi}{2}] \cup [\frac{\pi}{2}, \frac{3\pi}{2}] \cup [\frac{5\pi}{2}, \frac{7\pi}{2}] \cup \cdots$

Local min's: $(-\frac{\pi}{2}, -1), (\frac{3\pi}{2}, -1), \cdots$

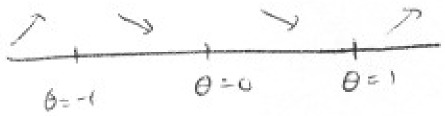
Local max's: $(-\frac{3\pi}{2}, 1), (\frac{\pi}{2}, 1), (\frac{5\pi}{2}, 1), \cdots$

c) $h(\theta) = \theta + \frac{1}{\theta} = \theta + \theta^{-1}$

$\Rightarrow h'(\theta) = 1 - \frac{1}{\theta^2} = 0 \Rightarrow 1 = \frac{1}{\theta^2}$

$\Rightarrow \theta = \pm\sqrt{1} = \pm 1$

$h'(\theta)$ DNE if $\theta = 0$



$h'(-2) > 0$

$h'(-\frac{1}{2}) < 0$

$h'(\frac{1}{2}) < 0$

$h'(2) > 0$

Increasing: $(-\infty, -1] \cup [1, \infty)$

Decreasing: $[-1, 0) \cup (0, 1]$

Local min: $(1, 2)$

Local max: $(-1, 0)$

2) $f(x) = x^{5/3} - 5x^{2/3}$

a) Domain: $(-\infty, \infty)$

b) Intercepts: $x = 0 \Rightarrow f(0) = 0$ & x-and y-intercept

$f(x) = 0 \Rightarrow 0 = x^{5/3} - 5x^{2/3}$

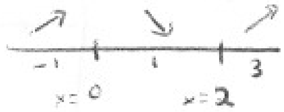
$= x^{2/3}(x-5) \Rightarrow (5, 0)$ is also an x-intercept

c) Asymptotes: None

d) $f'(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5x^{2/3}}{3} - \frac{10}{3x^{1/3}}$

$= \frac{5x - 10}{3x^{1/3}} = 0$ if $x = 2$

DNE if $x = 0$



Increasing: $(-\infty, 0] \cup [2, \infty)$

e) Decreasing: $[0, 2]$

f) Local Min: $(2, -3\sqrt[3]{4})$

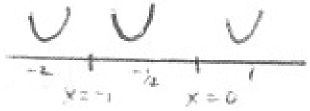
Local Max: $(0, 0)$

g) $f''(x) = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3}$

$= \frac{10}{9x^{1/3}} + \frac{10}{9x^{4/3}} = \frac{10x + 10}{9x^{4/3}}$

$= \frac{10(x+1)}{9x^{4/3}} = 0$ if $x = -1$

DNE if $x = 0$



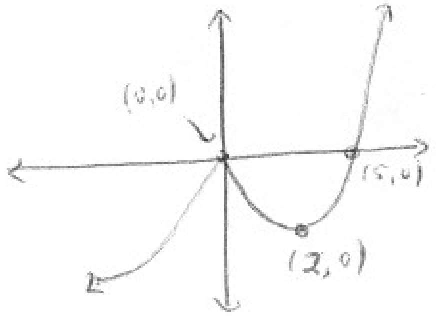
Concave upward: $(-\infty, \infty)$

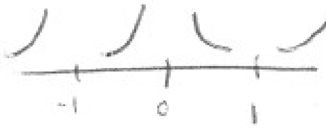
Concave Downward: Nowhere

h) POI: $(-1, -6)$ and $(0, 0)$

i) $f(-x) = -x^{5/3} - 5x^{2/3}$ (No Symmetry)

j)

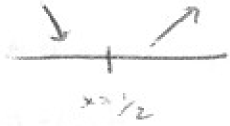




3) $m_{tan} = y' = 6x^2 - 6x + 6$

we want to minimize y' on $(-\infty, \infty)$

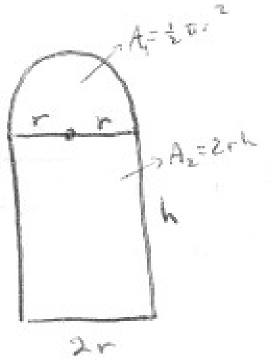
$y'' = 12x - 6 = 6(2x-1) = 0 \Rightarrow x = \frac{1}{2}$



Local min @ $x = \frac{1}{2}$, only critical # $\Rightarrow$ Absolute min

Slope: $y'\big|_{x=\frac{1}{2}} = 6(\frac{1}{2})^2 - 6(\frac{1}{2}) + 6 = 4.5$

4)



$A = \frac{1}{2}\pi r^2 + 2rh$

Perimeter of semicircle

Perimeter $= 2r + 2h + \pi r = 30$

$\Rightarrow h = 15 - r(1 + \frac{\pi}{2})$

Hence $A = \frac{1}{2}\pi r^2 + 2r(15 - r(1+\frac{\pi}{2}))$ maximize A for $r \geq 0$

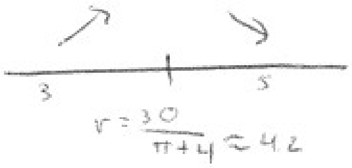
$= \frac{1}{2}\pi r^2 + 30r - 2r^2(1 + \frac{\pi}{2})$

$\Rightarrow A' = \pi r + 30 - 4r(1 + \frac{\pi}{2})$

$= r(\pi - 4 - 2\pi) + 30$

$= r(-4 - \pi) + 30 = 0$

$\Rightarrow r(\pi + 4) = 30 \Rightarrow r = \frac{30}{\pi + 4}$



Local max at $r = \frac{30}{\pi+4}$, only critical #

$\Rightarrow$ Absolute max @ $r = \frac{30}{\pi+4} \Rightarrow h = 15 - \frac{30}{\pi+4}(1 + \frac{\pi}{2})$

$= \frac{\cancel{15\pi} + 60 - 30 - \cancel{15\pi}}{\pi + 4}$

$= \frac{30}{\pi+4}$

Dimensions: Height $= \frac{30}{\pi+4}$

width $= 2(\frac{30}{\pi+4}) = \frac{60}{\pi+4}$

5) x = # passengers

$R(x)$ = revenue with x passengers

$$R(x) = \begin{cases} 10x & \text{if } x \le 200 \\ 10x - (x-200)(.02)x & \text{if } x > 200 \end{cases}$$

$$= \begin{cases} 10x & \text{if } x \le 200 \\ -.02x^2 + 14x & \text{if } x > 200 \end{cases}$$

Maximize $R(x)$ on $[0, 450]$

$$R'(x) = \begin{cases} 10 & \text{if } x < 200 \\ -.04x + 14 & \text{if } x > 200 \end{cases}$$

$$= 0 \quad \text{if} \quad x = \frac{14}{.04} = 350$$

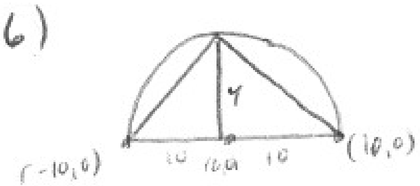
Since $R(x)$ is continuous on $[0, 450]$, the absolute max + min must occur at an endpoint or critical #.

$R(0) = 0$

$R(350) = 2450$ Maximum revenue occurs with

$R(450) = 2{,}250$ 350 passengers.

6)



We want to maximize

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}(20)y = 10y$$

, we can observe by inspection that the maximum area will be when $y = 10$

Equation of semicircle: $x^2 + y^2 = 10^2$ $(y \ge 0)$

$$\Rightarrow y = \sqrt{10^2 - x^2} = \sqrt{100 - x^2}$$

maximize $A = 10\sqrt{10^2 - x^2}$ on $[-10, 10]$

$$\Rightarrow A' = \frac{-20x}{2\sqrt{100-x^2}} = \frac{-10x}{\sqrt{100-x^2}} = 0 \text{ if } x = 0$$

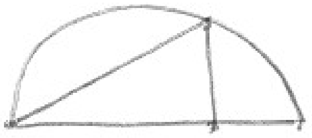
DNE if $x = \pm 10$

$A(-10) = A(10) = 0$

$A(0) = 10(10) = 100$ ← maximum

Dimensions: $\sqrt{200} \times \sqrt{200} \times 20$

Problem may also be read as



The answer would of course be different