Spring '09

Math 125 - Practice Exam 2

1a) $f(x) = (2x^4 - x^{-1/2})(-9x - e^{x^2} + 13)$

$\Rightarrow f'(x) = (2x^4 - x^{-1/2})(-9 - 2xe^{x^2}) + (8x^3 + \frac{1}{2}x^{-3/2})(-9x - e^{x^2} + 13)$

b) $f(x) = \dfrac{x^6 - x}{2 - \cos(3x)}$

$\Rightarrow f'(x) = \dfrac{(2-\cos(3x))(6x^5 - 1) - (x^6 - x)(3\sin(3x))}{(2-\cos(3x))^2}$

c) $f(x) = \sqrt[5]{(x^2+6x)^3} = (x^2+6x)^{3/5}$

$\Rightarrow f'(x) = \frac{3}{5}(x^2+6x)^{-2/5}(2x+6)$

d) $f(x) = \sin(\cos(7x^3))$

$\Rightarrow f'(x) = \cos(\cos(7x^3)) \cdot (-\sin(7x^3)) \cdot 21x^2$

$= -21x^2 \cdot \cos(\cos(7x^3)) \cdot \sin(7x^3)$

2a) $f(x) = 7\ln(x^8 + 2x)$

$\Rightarrow f'(x) = \dfrac{7(8x^7+2)}{x^8+2x}$

b) $f(x) = \ln\left(\dfrac{(x^2+3)^8}{(1-x)^{1/2}}\right) = \ln(x^2+3)^8 - \ln(1-x)^{1/2}$

$= 8\ln(x^2+3) - \frac{1}{2}\ln(1-x)$

$\Rightarrow f'(x) = \dfrac{8(2x)}{x^2+3} - \dfrac{-1}{2(1-x)}$

$= \dfrac{16x}{x^2+3} + \dfrac{1}{2(1-x)}$

c) $f(x) = (\ln(x^5 - x))^3$

$\Rightarrow f'(x) = 3(\ln(x^5-x))^2 \cdot \left(\dfrac{5x^4 - 1}{x^5 - x}\right)$

3) $f(x) = e^{\cos(7x)} + \sin^5(x + e^x + x^5)$

$$\Rightarrow f'(x) = -7\sin(7x)\cdot e^{\cos(7x)} + 5(\sin^4(x+e^x+x^5))\cos(x+e^x+x^5 \cdot (1+e^x+5x^4))$$

$$= -7e^{\cos(7x)}\sin(7x) + 5(1+e^x+5x^4)\cdot\sin^4(x+e^x+x^5)\cdot\cos(x+e^x+x^5).$$

4) $f(x) = \frac{4}{\sqrt{3x-21}} = 4(3x-21)^{-1/2}$

$$\Rightarrow f'(x) = -2(3x-21)^{-3/2}(3)$$

$$= -6(3x-21)^{-3/2}$$

$$\Rightarrow f''(x) = 9(3x-21)^{-5/2}(3)$$

$$= 27(3x-21)^{-5/2}$$

5) $f(x) = \cot x = \frac{\cos x}{\sin x}$

$\Rightarrow$ By Quotient Rule, $f'(x) = \frac{(\sin x)(\cos x)' - (\cos x)(\sin x)'}{(\sin x)^2}$

$$= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$= -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x.$$

6) $f(x) = \cos^{-1}(2x+1)$

$$\Rightarrow f'(x) = \frac{-2}{\sqrt{1-(2x+1)^2}}$$

7. $s(t) = t^3 - 6t^2 + 9t$

a) $v(t) = s'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$

$= 3(t-1)(t-3) = 0$

if $t = 1\,s$ or $t = 3\,s$

$a(t) = v'(t) = s''(t) = 6t - 12$

$a(1) = 6 - 12 = -6\ m/s^2$

$a(3) = 18 - 12 = 6\ m/s^2$

b) $a(t) = 6t - 12 = 0 \Rightarrow t = 2\,s$

$|v(2)| = |3(2)^2 - 12(2) + 9|$

$= |12 - 24 + 9|$

$= |-3|$

$= 3\ m/s$

c) $v(t) = 3(t-1)(t-3)$

$v(0) > 0$ $\quad v(2) < 0$ $\quad v(4) > 0$

$+ \quad - \quad +$

$1 \quad 3$

moving forward: $0 \le t < 1$ or $t > 3$

moving backward: $1 < t < 3$

8) $f(x) = x^3 \tan^{-1}\sqrt{x^2+1}$

$\Rightarrow f'(x) = 3x^2 \tan^{-1}\sqrt{x^2+1} + x^3 \cdot \dfrac{1}{1+(\sqrt{x^2+1})^2} \cdot \dfrac{1}{\cancel{2}}(x^2+1)^{-1/2} \cdot (\cancel{2}x)$

$= 3x^2 \tan^{-1}\sqrt{x^2+1} + \dfrac{x^4}{x^2+2} \cdot \dfrac{1}{\sqrt{x^2+1}}$

9) $S = 2\pi r^2 + 2\pi r h$

a) $\dfrac{dS}{dt} = 4\pi r \dfrac{dr}{dt} + 2\pi h \dfrac{dr}{dt} + 2\pi r \overbrace{\dfrac{dh}{dt}}^{=0 \text{ if } h \text{ is constant}}$

$= 2\pi(2r + h)\dfrac{dr}{dt}$

b) $\dfrac{dS}{dt} = 2\pi(2r + h)\dfrac{dr}{dt} + 2\pi r \dfrac{dh}{dt}$

10 a) $(4f(x)+9g(x))' = 4f'(x)+9g'(x)$

$\Rightarrow 4f'(-1)+9g'(-1) = 4\left(\frac{-1}{3}\right)+9(1)$

$= \frac{23}{3}$

b) $\left(\sqrt{f(x)}\cdot g(x)\right)' = \frac{1}{2}f(x)^{-1/2}\cdot f'(x)\cdot g(x) + \sqrt{f(x)}\cdot g'(x)$

$\Rightarrow \left(\sqrt{f(5)}\cdot g(5)\right)' = \frac{1}{\cancel{2}\sqrt{4}}\cdot \cancel{(2)}(3)+\sqrt{4}\cdot(-7)$

$= \frac{3}{2} - 14$

$= \frac{-25}{2}$

c) $[g(f(x))]' = g'(f(x))\cdot f'(x)$

$\Rightarrow [g(f(-1))]' = g'(5)\cdot\frac{-1}{3}$

$= (-7)\left(\frac{-1}{3}\right)$

$= \frac{7}{3}$

d) $\left[\left(x^3+g(x)\right)^2\right]' = 2(x^3+g(x))\cdot(3x^2+g'(x))$

$\Rightarrow$ @ $x=-1$, we have $2(-1+11)\cdot(3+1) = 2(10)(4)$

$= 80$

11) $2x - 20x^3y^2 - 10x^4y\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 2x - 20x^3y^2 = \frac{dy}{dx}(10x^4y+3y^2)$

$\Rightarrow \frac{dy}{dx} = \frac{2x-20x^3y^2}{10x^4y+3y^2}$

12) $x\frac{dy}{dx} + y + 2 - 5\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}(x-5) = -y-2$

$$\Rightarrow \frac{dy}{dx} = \frac{-y-2}{x-5}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(3,2)} = \frac{-2-2}{3-5}$$

$$= \frac{-4}{-2}$$

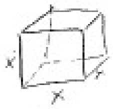
$$= 2$$

Tangent Line: $y - 2 = 2(x-3)$

$$\Rightarrow y - 2 = 2x - 6$$

$$\Rightarrow y = 2x - 4$$

13) $V = x^3$



$\frac{dV}{dt} = 1200\ cm^3/min$

$x = 20\ cm$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

$$1200 = 3(20)^2\frac{dx}{dt}$$

$$\Rightarrow \cancel{1200} = \cancel{1200}\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = 1\ cm/min$$

14a) $y' = \frac{1}{(e^{-t})^2+1} \, (e^{-t})' \ldots$

$= \frac{-e^{-t}}{e^{-2t}+1}$

b) $f(x) = \tan^{-1}\sqrt{x^2-1} + \sin^{-1}(3x)$

$\Rightarrow f'(x) = \frac{1}{(\sqrt{x^2-1})^2+1} \cdot \frac{1}{\cancel{2}}(x^2-1)^{-1/2}(\cancel{2}x) + \frac{1}{\sqrt{1-(3x)^2}}(3)$

$= \frac{x}{\sqrt{x^2-1}\,[(x^2-\cancel{1})+\cancel{1}]} + \frac{3}{\sqrt{1-9x^2}}$

$= \frac{1}{x\sqrt{x^2-1}} + \frac{3}{\sqrt{1-9x^2}}$

15) $f(x) = e^{-x}$

-0.1 is "near" $a = 0$

$f(0) = e^0 = 1$

$f'(x) = -e^{-x} \Rightarrow f'(0) = -1$

$(L(x) = f(a) + f'(a)(x-a))$

$L(x) = 1 + (-1)(x-0)$

$= 1 - x$

$\Rightarrow e^{-0.1} \approx L(-0.1) = 1 - (-0.1) = \boxed{1.1}$

(Note: $f(-.1) = e^{-(-.1)} \approx 1.105$)

16 a) $f(x) = 7x^3 - 6x^{2/3} - 5x^{-4} + e^{+x} + 5\pi \Rightarrow f'(x) = 21x^2 - 4x^{-1/3} + 20x^{-5} + e^x$

b) $f(x) = 2x + x^2e^x \Rightarrow f'(x) = 2 + 2xe^x + x^2e^x$

$= 2 + xe^x(2+x)$

c) $f(x) = (x^3 - 5x^{1/2})(4x^2 + 3x^8 + e^x) = 4x^5 + 3x^{11} + x^3e^x - 5x^{5/2} - 15x^{17/2} - 5x^{1/2}e^x$

$\Rightarrow f'(x) = 20x^4 + 33x^{10} + 3x^2e^x + x^3e^x - \frac{25}{2}x^{3/2} - 255x^{15/2} - \frac{5}{2}x^{-1/2}e^x - 5x^{1/2}e^x$

d) $f(x) = \frac{t^2-1}{t^2+1} \Rightarrow f'(x) = \frac{(t^2+1)(2t) - (t^2-1)(2t)}{(t^2+1)^2} = \frac{\cancel{2t^3} + 2t - \cancel{2t^3} + 2t}{(t^2+1)^2}$

$= \frac{4t}{(t^2+1)^2}$