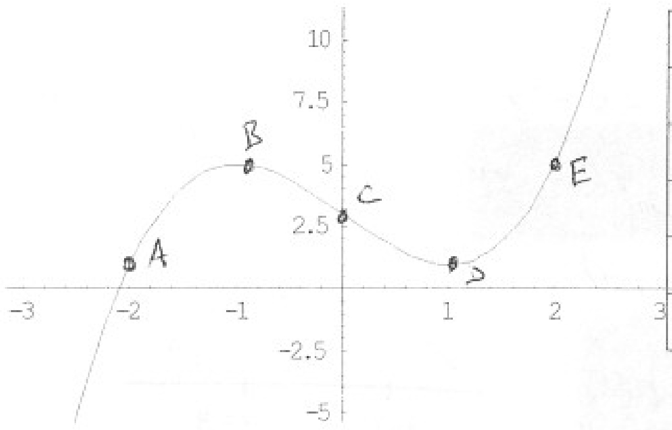
Math 125.01
Practice Test 3

1. Using the graph of $f(x)$ below, determine if $f(x)$, $f'(x)$, and $f''(x)$ at each marked point is positive, negative, or zero.



	$f(x)$	$f'(x)$	$f''(x)$
A	+	+	−
B	+	0	−
C	+	−	0
D	+	0	+
E	+	+	+

2. Let $f(x)=\dfrac{x^4}{2}-3x^2,\ -4\le x\le 1.$

a) Identify the function's local extreme values in the given domain, and say where they are assumed.

$f'(x) = 2x^3 - 6x$

$= 2x(x^2-3)=0$

$\Rightarrow x = 0$ or

$x = \pm\sqrt{3}$

min max

$x=-4$ $\quad -3 \quad x=-\sqrt{3} \quad -1 \quad x=0 \quad \tfrac{1}{2} \quad x=1$

$2x(x^2-3)$

$x=-3 \quad - \cdot + = -$

$x=-1 \quad - \quad - = +$

$x=\tfrac{1}{2} \quad + \quad - = -$

local max: $(-4, 80)$ and $(0,0)$

local min: $(-\sqrt{3}, -4.5)$

b) Endpoints: $(-4, 80)$ and $(1, -2.5)$

Abs. max value $= 80$ @ $x=-4$

Abs. min value $= -4.5$ @ $x=-\sqrt{3}$

3) $f(x) = x\sqrt{3-x}$ on $[-1,3]$

a) $f'(x) = \sqrt{3-x} - \frac{1}{2}x(3-x)^{-1/2} = 0$ $\quad$ DNE @ $x=3$

$= \dfrac{2(3-x)-x}{2\sqrt{3-x}} = \dfrac{6-3x}{2\sqrt{3-x}}$

$\Rightarrow \sqrt{3-x} = \dfrac{x}{2\sqrt{3-x}} \Rightarrow 2(3-x)=x \Rightarrow 6=3x \Rightarrow x=2$

$x=-1 \quad x=2 \quad x=3$

local and absolute max: $(2,2)$

local min: $(-1,-2)$ and $(3,0)$

absolute min: $(-1,-2)$

4) $f(-1) = (-1)^2 - 5(-1) + 4 = 10$ $\Rightarrow$ $\dfrac{f(3) - f(-1)}{3 - -1} = \dfrac{-2 - 10}{4} = \dfrac{-12}{4} = {}^{-}3$

$f(3) = (3)^2 - 5(3) + 4 = -2$

MVT $\Rightarrow$ There is some $c \in (-1, 3)$ such that $f'(c) = {}^{-}3$

$f'(x) = 2x - 5$

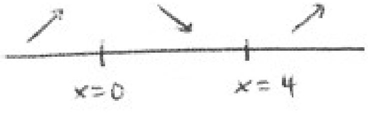
Hence $f'(c) = {}^{-}3 \Rightarrow 2c - 5 = {}^{-}3$

$\Rightarrow \boxed{c = 1}$

5) $f(x) = x^5 - 5x^4$ Domain: $(-\infty, \infty)$

a) $f'(x) = 5x^4 - 20x^3 = 5x^3(x - 4) = 0$

if $x = 0, x = 4$

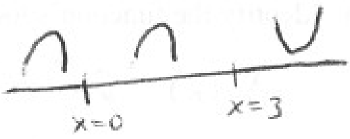


Increasing: $(-\infty, 0] \cup [4, \infty)$

Decreasing: $[0, 4]$

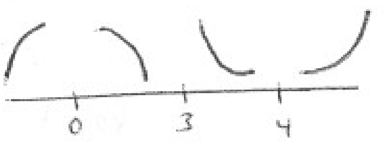
b) $f''(x) = 20x^3 - 60x^2 = 20x^2(x - 3) = 0$

if $x = 0, x = 3$



Concave Down: $(-\infty, 3]$

Concave Up: $[3, \infty)$

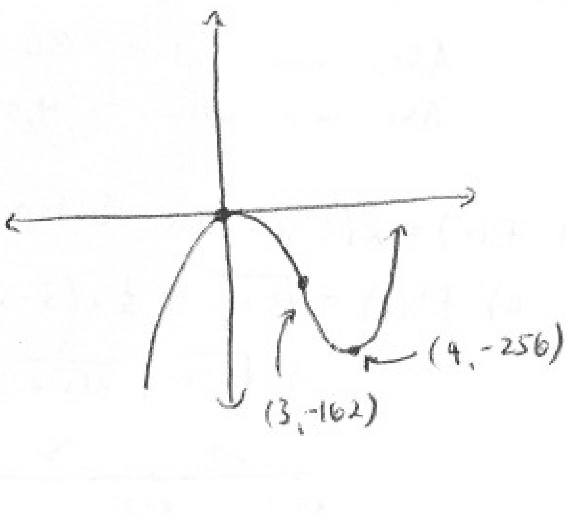


c) Inflection points: $(3, -162)$

d) Intercepts: $(0, 0)$

Local Max: $(0, 0)$

Local Min: $(4, -256)$



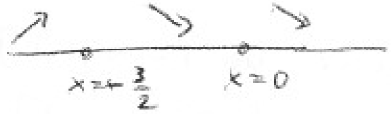
6) $f(x) = -x^4 - 2x^3 = -x^3(x+2)$: Domain: $(-\infty, \infty)$

a) $f'(x) = -4x^3 - 6x^2$

$= -2x^2(2x+3) = 0$

if $x = 0$

$x = -\frac{3}{2}$

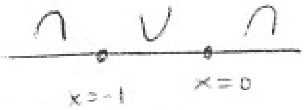


Increasing: $(-\infty, -\frac{3}{2}]$

Decreasing: $[-\frac{3}{2}, \infty)$

b) $f''(x) = -12x^2 - 12x = -12x(x+1) = 0$

if $x = 0$, $x = -1$



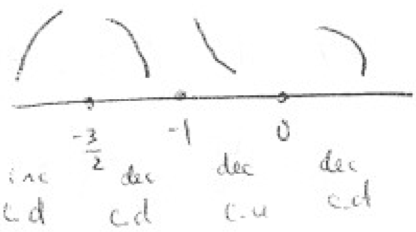
Concave Up: $[-1, 0]$

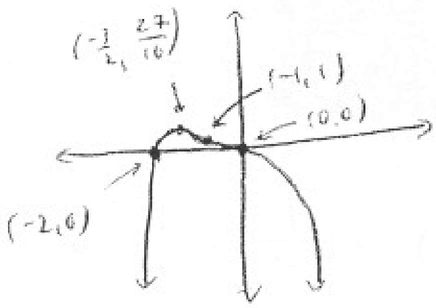
Concave Down: $(-\infty, -1] \cup [0, \infty)$

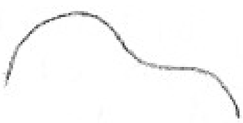
c) Inflection Points: $(-1, 1)$ and $(0, 0)$

Local Max: $(-\frac{3}{2}, \frac{27}{16})$

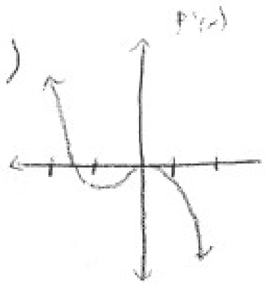
Intercepts: $(0, 0)$ and $(-2, 0)$







d)

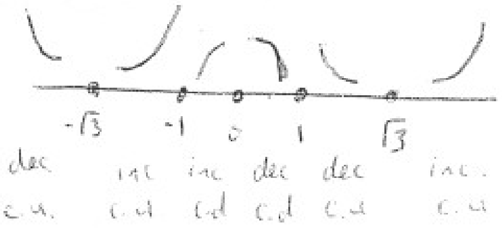


7a) Increasing: $[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)$

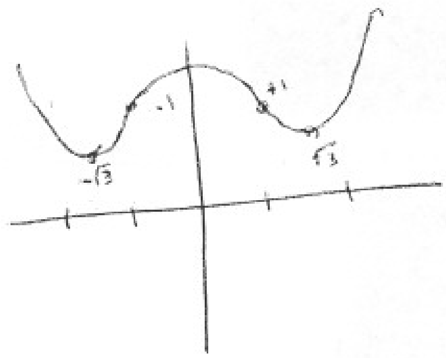
Decreasing: $(-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$

b) Concave up: $(-\infty, -1] \cup [1, \infty)$

Concave down: $[-1, 1]$



c)



8) $T = \left(\frac{C}{2} - \frac{D}{3}\right) D^2 = \frac{1}{6}(3C - 2D) D^2$

a) $\frac{dT}{dD} = -\frac{1}{3} D^2 + \frac{1}{6}(3C - 2D)(2D)$

$= -\frac{1}{3} D (D - 3C + 2D)$

$= -\frac{1}{3} D(3D - 3C) = 0$ if $D = 0$ or $D = C$

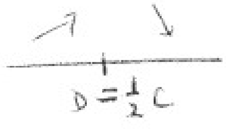
$= -D(D - C)$

Maximized when $D = C$

b) $\frac{dT}{dD} = -\frac{1}{2} D(D - C) \Rightarrow \frac{d^2T}{dD^2} = -\frac{1}{2}(D - C) - \frac{1}{2} D$

$= -D + \frac{1}{2} C = 0$

$\Rightarrow D = \frac{1}{2} C$

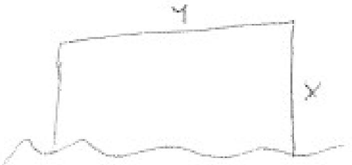


Sensitivity Maximized when $D = \frac{1}{2} C$

9a) $\lim_{x \to 5} \frac{x - 5}{x^2 - 25}$ $\frac{0}{0}$

(L'H) $= \lim_{x \to 5} \frac{1}{2x} = \frac{1}{10}$

b) $\lim_{x \to 3} \frac{x^2 + 5x + 6}{x + 3} = \frac{30}{6} = 5$

10) 

Perimeter $= \frac{5700}{5} = 1140$ ft

$2x + y = 1140$

Maximize $A = xy = x(1140-2x)$ $\quad x \in [0, 57$

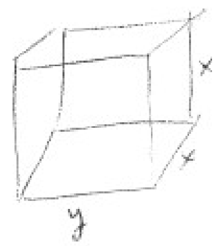
$= 1140x - 2x^2$

$A'(x) = 1140 - 4x = 0$ $\quad \nearrow \; \searrow \quad x = 285$

$\Rightarrow x = \frac{1140}{4} = 285$

$(y = 1140 - 2(285) = 570)$ local max & abs. max (only 1 critical #)

Dimensions: 285 ft × 570 ft

11) 

$V = x^2 y = 4500 \text{ in}^3 \Rightarrow y = \frac{4500}{x^2}$

minimize S. A $= xy + 2x^2 + 2xy$

$= 3xy + 2x^2$

$= 3x\left(\frac{4500}{x^2}\right) + 2x^2$ $\quad (x > 0)$

$= 13500x^{-1} + 2x^2$

$\Rightarrow A' = \frac{-13500}{x^2} + 4x = 0$

$\Rightarrow 4x^3 = 13{,}500$

$\Rightarrow x^3 = 3{,}375$ $\quad \searrow \; \nearrow$

$\Rightarrow x = \sqrt[3]{3375} = 15$ in $\quad x=0 \quad x = 15$

$y = \frac{4500}{15^2} = 20$ $\quad$ local min @ $x = 15 \Rightarrow$ abs. min since only 1 critical #

Dimensions: 20 in × 15 in × 15 in