Math 224.02 Name:________________________________

Lab 1 - Review, Section 5.3

August 27, 2009

1. Find the equation of the tangent line to the function $f(x) = x^2 + 3$ at the point whose x-coordinate is $x = 1$.

2. Given the equation $y = 3x^4 - 4x^3 - 12x^2 + 2$, determine the following:

   (a) Intervals of Increase/Decrease

   (b) Local Extrema (Label which are local maxima and which are local minima)

   (c) Intervals on which the graph is concave upward/downward

   (d) Points of inflection

   (e) Sketch the curve

3. **Riemann Sums**

   Suppose we want to find the area of the region that lies under the curve $y = f(x)$ and above the x-axis from $x = a$ to $x = b$. That is, we want to find the area of the region bounded by the graph of a function f, the vertical lines a and b, and the $x-$axis. To do so, we approximate the region by rectangles and then take the limit of the areas of these rectangles. We start by subdividing the interval $[a, b]$ into n smaller subintervals by choosing partition points $x_0, x_1, x_2, ..., x_n$ so that $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. (To make things easier, we'll assume that each of these subintervals have the same width, Δx.)

   By drawing the lines $x = a, x = x_1, x = x_2, ..., x = b$, we use the partition P to divide the region into strips $S_1, S_2, ..., S_n$ and approximate these strips S_i by rectangles R_i. To do this we construct a rectangle R_i with base Δx and height $f(x_i)$. (Note: These are right sums. Why?) Then the area of the i^{th} rectangle R_i is $A_i =$ ____________. We can approximate the area of the region by summing the areas of the rectangles, which is $\sum_{i=1}^{n} A_i = \sum_{i=1}^{n}$ ____________. As $\Delta x \to 0$, the subintervals become more numerous and shorter at the same time. That is, the rectangles will increase in number and get less wide (although their total length is still $b - a$), and our approximation will become better. Thus we define the **area** A of the region as the limiting value (if it exists) of the areas of the approximating polygons. In symbols, this is $A = \lim_{\_\_\_\_\to 0} \sum_{i=1}^{n}$ ____________.

4. Find an antiderivative of each of the following.

   (a) $g(z) = \sqrt{z}$

   (b) $g(y) = y^4 + \dfrac{1}{y}$

   (c) $f(t) = \dfrac{t^2 + 1}{t}$

5. Determine the following indefinite integrals.

(a) $\int (x^2 + 5x + 8)\ dx$

(b) $\int e^{2r}\ dr$

(c) $\int \left(\frac{y^2 - 1}{y}\right)^2\ dy$

(d) $\int 3e^{3z} + \sin z + \cos z\ dz$

(e) $\int 3\cos 5\theta\ d\theta$

(f) $\int (4\sec x \tan x - 2\sec^2 x)\ dx$

(g) $\int (1 + \cot^2 x)\ dx$

6. Verify the formulas by differentiation.

(a) $\int \frac{1}{(x+1)^2}\ dx = -\frac{1}{x+1} + C$

(b) $\int xe^x\ dx = xe^x - e^x + C$

7. A graph of f is given below. Let $F'(x) = f(x)$.

(a) What are the critical points of $F(x)$?

(b) Which critical points are local maxima, which are local minima, and which are neither?

(c) Sketch a possible graph of $F(x)$.