Math 224.02 Name:______________________________

Lab 2 - Sections 5.4-5.6

September 3, 2009

1. Evaluate the following definite integrals exactly.

   (a) $\displaystyle\int_1^3 \frac{1}{t}\, dt$

   (b) $\displaystyle\int_0^2 \left(\frac{x^3}{3} + 2x\right)\, dx$

   (c) $\displaystyle\int_0^{\pi/4} \frac{1}{\cos^2 x}\, dx$

2. Find the exact value of the area between the graphs of $y = \cos x$ and $y = e^x$ for $0 \le x \le 1$.

3. Use the Fundamental Theorem to determine the value of b if the area under the graph of $f(x) = 8x$ between $x = 1$ and $x = b$ is equal to 192. Assume $b > 1$.

4. For each part, answer the following:

   (a) Determine the derivative of the function, $F'(x)$.

   (b) Note that $F(x)$ is an antiderivative of $F'(x)$.

   (i) $F(x) = e^{x^2+3x}$

   (ii) $F(x) = \dfrac{3}{x^2 + 5x + 7}$

   (iii) $F(x) = \sin(e^{5x})$

   (iv) $F(x) = \dfrac{2x + 1}{(x + 1)(3x + 5)}$

   (v) $F(x) = 2xe^x - 2e^x$

   (vi) $F(x) = \dfrac{1}{2}e^x(\sin x - \cos x)$

5. Evaluate the integrals, if they exist.

   (a) $\displaystyle\int \frac{2x + 3}{3x^2 + 9x + 7}\, dx$

   (b) $\displaystyle\int e^x \sin(e^x)\, dx$

   (c) $\displaystyle\int \frac{1}{x^2}\sqrt{1 + \frac{1}{x}}\, dx$

   (d) $\displaystyle\int \frac{\sin t}{(5 + \cos t)^4}\, dt$

   (e) $\displaystyle\int \frac{\ln 5x}{x}\, dx$

6. Evaluate the integrals, if they exist.

   (a) $\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\ dx$

   (b) $\int_0^{\pi/3} \frac{\sin\theta}{\cos^2\theta}\ dx$

   (c) $\int_0^1 x(x^2+2)^3\ dx$

7. If f is continuous on R, prove that

$$\int_a^b f(x+c)\ dx = \int_{a+c}^{b+c} f(x)\ dx.$$

8. If a and b are positive numbers, show that

$$\int_0^1 x^a(1-x)^b = \int_0^1 x^b(1-x)^a\ dx.$$