Lab 3

1a) $\int x \sin 4x \, dx$ $\quad u = x \quad du = dx \quad v = -\frac{1}{4}\cos x \quad dv = \sin 4x \, dx$

$= -\frac{1}{4}x \cos 4x + \frac{1}{4}\int + \cos 4x \, dx$

$= -\frac{1}{4}x \cos 4x + \frac{1}{16}\sin 4x + C$

b) $\int t^3 e^t \, dt$

$= t^3e^t - 3t^2e^t + 6te^t - 6e^t + C$

t^3	+	e^t
$3t^2$	−	e^t
$6t$	+	e^t
6	−	e^t
		e^t

c) $\int_1^4 \sqrt{t} \cdot \ln t \, dt$ $\quad u = \ln t \quad du = \frac{1}{t}dt \quad v = \frac{2}{3}t^{3/2} \quad dv = \sqrt{t}\, dt = t^{1/2}dt$

$= \frac{2}{3}t^{3/2}\ln t \Big|_1^4 - \int_1^4 \frac{2}{3}t^{3/2} \cdot \frac{1}{t}dt$

$= \frac{2}{3} \cdot 8 \cdot \ln 4 - 0 - \frac{2}{3}\int_1^4 t^{1/2}dt$

$= \frac{16}{3}\ln 4 - \frac{2}{3} \cdot \frac{2}{3}t^{3/2}\Big|_1^4$

$= \frac{16}{3}\ln 4 - \frac{4}{9}(8) + \frac{4}{9} = \frac{16}{3}\ln 4 - \frac{28}{9}$

d) $\int \frac{e^x+1}{e^x}dx = \int 1 + e^{-x}dx = x - e^{-x} + C$

f) $\int_1^4 \ln \sqrt{x}\, dx = \frac{1}{2}\int_1^4 \ln x \, dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx \quad v = x \quad dv = dx$

Note: this is out of order

$= \frac{1}{2}\left[x \ln x \Big|_1^4 - \int_1^4 x \cdot \frac{1}{x}dx\right]$

$= \frac{1}{2}\left(4 \ln 4 - 0 - (x\Big|_1^4\right.$

$= \frac{1}{2}(4\ln 4 - (4-1))$

$= 2\ln 4 - \frac{3}{2}$

e) $\int \frac{x+1}{x^2+2x}\,dx = \int \frac{x+1}{x(x+2)}\,dx$

$\frac{x+1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$\Rightarrow A = \frac{1}{2}$

$B = \frac{-1}{-2} = \frac{1}{2}$

(Heaviside Cover-Up)

$= \int \frac{1/2}{x} + \frac{1/2}{x+2}\,dx$

$= \frac{1}{2}\ln|x| + \frac{1}{2}\ln|x+2| + C$

g) $\int \frac{\ln 3x}{x}\,dx$

$u = \ln 3x$

$du = \frac{1}{3x}\cdot 3dx = \frac{1}{x}dx$

$= \int u\,du$

$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln 3x)^2 + C$

h) $\int \sin(\ln x)dx$

$u = \sin(\ln x)$ $\quad v = x$

$du = \frac{\cos(\ln x)}{x}dx$ $\quad dv = dx$

$= x\sin(\ln x) - \int \cancel{x}\,\frac{\cos(\ln x)}{\cancel{x}}\,dx$

$u = \cos(\ln x)$ $\quad v = x$

$du = \frac{-\sin(\ln x)}{x}dx$ $\quad dv = dx$

$= x\sin(\ln x) - \left(x\cos(\ln x) - \int \frac{-\sin(\ln x)}{\cancel{x}}\,\cancel{x}dx\right)$

$\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)dx$

$\Rightarrow 2\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x)$

$\Rightarrow \int \sin(\ln x)dx = \frac{1}{2}\left[x\sin(\ln x) - x\cos(\ln x)\right] + C$

i) $\int x^5 e^{x^2}dx = \int (x^2)^2 \cdot e^{x^2} \cdot x\,dx$

$w = x^2$

$dw = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int \frac{1}{2}w^2 e^w\,dw$

$= \frac{1}{2}\int w^2 e^w\,dw$

w^2 +	e^w
$2w$ −	e^w
2 +	e^w
	e^w

$= \frac{1}{2}\left(w^2e^w - 2we^w + 2e^w\right) + C$

$= \frac{1}{2}w^2e^w - we^w + e^w + C$

$= \frac{1}{2}x^4e^{x^2} - x^2e^{x^2} + e^{x^2} + C$

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1j) $\int \frac{e^{\tan x}}{\cos^2 x} dx$

$u = \tan x$

$du = \sec^2 x\, dx = \frac{1}{\cos^2 x} dx$

$= \int e^u du$

$= e^u + C = e^{\tan x} + C$

k) $\int_1^4 e^{\sqrt{x}} dx$

$w = \sqrt{x}$

$dw = \frac{1}{2\sqrt{x}} dx$

$= \frac{1}{2w} dx \Rightarrow 2w\,dw = dx$

$x = 1 \Rightarrow w = \sqrt{1} = 1$

$x = 4 \Rightarrow w = \sqrt{4} = 2$

$= \int_1^2 2we^w dw$

$= 2\int_1^2 we^w dw$

Now, Integration by parts

$u = w \qquad v = e^w$

$du = dw \qquad dv = e^w dw$

$= 2\left[we^w \Big|_1^2 - \int_1^2 e^w dw \right]$

$= 2\left[2e^2 - e - e^w \Big|_1^2 \right]$

$= 2\left[2e^2 - \cancel{e} - e^2 + \cancel{e} \right]$

$= 2e^2$

l) $\int \frac{\sin x \cdot \cos x}{1 - \cos^2 x} dx$

$u = 1 - \cos^2 x$

$du = -2\cos x \cdot \frac{d}{dx}(\cos x)$

$= 2\cos x \sin x\, dx$

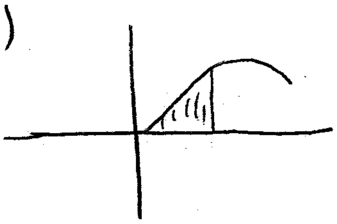
$\Rightarrow \frac{1}{2} du = \sin x \cdot \cos x\, dx$

$= \int \frac{\frac{1}{2} du}{u}$

$= \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|1 - \cos^2 x| + C$

2a)



$A = \int_0^{\pi/3} \sin\left(\frac{x}{2}\right) dx = -2\cos\left(\frac{x}{2}\right) \Big|_0^{\pi/3}$

$= -2\left(\cos\frac{\pi}{6} - \cos 0\right)$

$= -2\left(\frac{\sqrt{3}}{2} - 1\right)$

$= \cancel{2}\left(\frac{2-\sqrt{3}}{\cancel{2}}\right)$

$= 2 - \sqrt{3}$

2b)

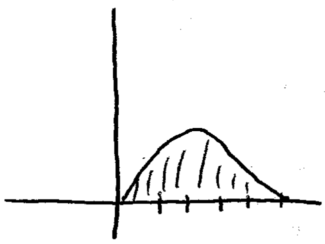
$$A = \int_1^2 x(x^2+1)^4\,dx \qquad u = x^2+1 \quad du = 2x\,dx \quad \tfrac{1}{2}du = x\,dx \qquad x=1 \Rightarrow u=2 \quad x=2 \Rightarrow u=5$$

$$= \int_2^5 \tfrac{1}{2} u^4\,du$$

$$= \tfrac{1}{10} u^5 \Big|_2^5$$

$$= \tfrac{1}{10}\left(5^5 - 2^5\right) = \frac{3093}{10}$$

c)



$$A = \int_0^5 x e^{-x}\,dx \qquad u = x \quad du = dx \qquad v = -e^{-x}dx \quad dv = e^{-x}dx$$

$$= -xe^{-x}\Big|_0^5 - \int_0^5 -e^{-x}\,dx$$

$$= -5e^{-5} + 0 - e^{-x}\Big|_0^5$$

$$= -5e^{-5} - e^{-5} + 1$$

3) $\int (\ln x)^n\,dx$

$$u = (\ln x)^n \quad du = n(\ln x)^{n-1} \cdot \tfrac{1}{x}dx \qquad v = x \quad dv = dx$$

$$= x(\ln x)^n - \int \cancel{x} \cdot n(\ln x)^{n-1} \cdot \tfrac{1}{\cancel{x}}\,dx$$

$$= x(\ln x)^n - \int n(\ln x)^{n-1}dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$$

4) $\int f(x)\,dx$

$$u = f(x) \quad du = f'(x)\,dx \qquad v = x \quad dv = dx$$

$$= x f(x) - \int x f'(x)\,dx$$

5 a) $$\frac{3}{x-2} - \frac{2}{x+5} = \frac{3(x+5) - 2(x-2)}{(x-2)(x+5)} = \frac{3x+15-2x+4}{(x-2)(x+5)} = \frac{x+19}{(x-2)(x+5)}$$

b) $$\frac{4}{(x-1)} + \frac{2}{(x+1)} + \frac{3}{(x+1)^2} = \frac{4(x+1)^2}{(x-1)(x+1)^2} + \frac{2(x-1)(x+1)}{(x-1)(x+1)^2} + \frac{3(x-1)}{(x-1)(x+1)^2}$$

$$= \frac{(4x^2+8x+4) + (2x^2-2) + (3x-3)}{(x-1)(x+1)^2}$$

$$= \frac{6x^2+11x-1}{(x-1)(x+1)^2}$$

c) $$\frac{2}{x-5} + \frac{3x+2}{x^2+1} = \frac{2(x^2+1) + (3x+2)(x-5)}{(x-5)(x^2+1)} = \frac{2x^2+2+3x^2-15x+2x-10}{(x-5)(x^2+1)} = \frac{5x^2-13x-8}{(x-5)(x^2+1)}$$

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6) $\frac{A}{x-4} + \frac{B}{x+2} = \frac{2x+1}{(x-4)(x+2)}$

$\Rightarrow A(x+2) + B(x-4) = 2x+1$

$x = -2 \Rightarrow -6B = -3 \Rightarrow B = \frac{1}{2}$

$x = 4 \Rightarrow 6A = 9 \Rightarrow A = \frac{3}{2}$

7) $\int \frac{x}{\sqrt{4-x^2}} dx$

$u = 4 - x^2$

$du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

$= \int \frac{-\frac{1}{2} du}{\sqrt{u}}$

$= -\frac{1}{2} \int u^{-1/2} du$

$= -\frac{1}{\cancel{2}} \cancel{2} u^{1/2} + C$

$= -\sqrt{4-x^2} + C$