

1. Write the word or phrase that best completes each statement or answers the question. Provide an appropriate response.

(a) Timothy wishes to evaluate the integral over all real numbers of

$$f(x) = \begin{cases} \frac{1}{x}, & \text{if } x < 0 \\ \frac{1}{x^2}, & \text{if } x > 0 \end{cases},$$

and claims this is zero because $-\infty + \infty$ equals zero. What is wrong with this thinking?

- (b) Stephen claims that integral $\int_a^b f(x) dx$ always exists, as long as a and b are both positive and finite. Refute this by giving an example of a function for which this is not true.
- (c) Maria knows that integral $\int_a^\infty f(x) dx$ diverges, but needs to investigate $\int_a^\infty g(x) dx$, where $g(x) = \frac{f(x)}{64}$. Does this integral necessarily also diverge?
- (d) Elizabeth knows that integral $\int_a^\infty f(x) dx$ converges. Does $\int_{-\infty}^a f(x) dx$ also necessarily converge?
2. Mary wishes to find the integral $\int_0^\infty f(x) dx$ of a function for which $\lim_{x \rightarrow \infty} f(x) = 1$. Why can this not be done?
3. Use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence.

(a) $\int_{-1}^1 \ln|x| dx$

(b) $\int_1^\infty \frac{dx}{x^3 + 1}$

(c) $\int_\pi^\infty \frac{2 + \cos x}{x} dx$

(d) $\int_1^\infty \frac{\sqrt{x+1}}{x^2} dx$

(e) $\int_1^\infty \frac{e^x}{x} dx$

(f) $\int_1^\infty \frac{x}{e^x} dx$

4. Find an upper bound for the error in estimating $\int_2^4 \frac{1}{x-1} dx$ using Simpson's Rule with $n = 8$ steps.

5. Given that we know the Fundamental Theorem of Calculus, why would we want to develop numerical methods for definite integrals?
- i. Antiderivatives are not always expressible in closed form.
 - ii. Numerical methods are a good excuse to use our graphing calculator.
 - iii. The function $f(x)$ may not be continuous on $[a, b]$.
- (a) Only iii is correct. (b) Only ii is correct. (c) Both i and iii are correct.
 (d) Only i is correct.
6. When we use Simpson's rule to approximate a definite integral, it is necessary that the number of partitions be
- (a) an even number. (b) either an even or odd number. (c) a multiple of 4.
 (d) an odd number.
7. The error formula for Simpson's Rule depends upon
- i. $f(x)$.
 - ii. $f''(x)$.
 - iii. $f^{(4)}(x)$
 - iv. the number of steps
- (a) ii and iv (b) i and iii (c) i, iii, and iv (d) iii and iv
8. The "Simpson" sum is based upon the area under a
- (a) parabola (b) trapezoid (c) triangle (d) rectangle
9. The error formula for the Trapezoidal Rule depends upon
- i. $f(x)$.
 - ii. $f'(x)$.
 - iii. $f''(x)$.
 - iv. the number of steps
- (a) iii and iv (b) i and iii (c) ii and iv (d) i, iii, and iv