Lab 5

1a) $-\infty + \infty$ is an indeterminant form (we need more information to know how it simplifies)

In this case, $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$, which diverges by definition since $\int_{-\infty}^{0} \frac{1}{x}\,dx$ diverges.

b) $\int_1^3 \frac{1}{x-2}\,dx$ diverges, as $\int_1^2 \frac{1}{x-2}\,dx = \lim_{b\to 2^-} \int_1^b \frac{1}{x-2}\,dx$

$$= \lim_{b\to 2^-} \ln|x-2| \Big|_1^b$$

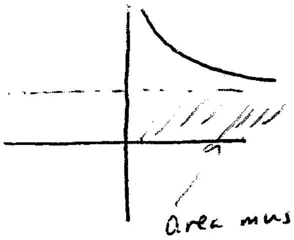
$$= \lim_{b\to 2^-} \ln|b-2| - \ln 1$$

$$= -\infty \quad \text{(Diverges)}$$

c) $\int_a^\infty g(x)\,dx = \int_a^\infty \frac{f(x)}{64}\,dx = \frac{1}{64}\int_a^\infty f(x)\,dx$, which diverges, so $\int_a^\infty g(x)\,dx$ diverges.

d) No – $\int_1^\infty \frac{1}{x^2}\,dx$ converges, but $\int_{-\infty}^{1} \frac{1}{x^2}\,dx$ diverges, since $\int_0^1 \frac{1}{x^2}dx$ diverges.

2)



area must be infinite

The only way that $\int_0^\infty f(x)$ can possibly converge is if $\lim_{x\to\infty} f(x) = 0$.

3a) $\int_{-1}^{1} \ln|x|\,dx = 2\int_0^1 \ln|x|\,dx$ (Since $\ln|x|$ is even-symmetry)

$$= 2\lim_{b\to 0^+} \int_b^1 \ln x\,dx \qquad \begin{matrix} u = \ln x & v = x \\ du = \frac{1}{x}dx & dv = dx \end{matrix}$$

$$= 2\lim_{b\to 0^+} x\ln x \Big|_b^1 - \int_b^1 x\cdot\frac{1}{x}dx$$

$$= 2\left(\lim_{b\to 0^+} b\ln b - \lim_{b\to 0^+} x\Big|_b^1\right)$$

$$= 2\left(\lim_{b\to 0^+} b\ln b - 1 + 0\right) = 2(-1) \quad \text{(Converges)}$$

$$= -2$$

$$\lim_{b\to 0^+} \frac{\ln b}{\frac{1}{b}} = \lim_{b\to 0^+} \frac{\frac{1}{b}}{\frac{-1}{b^2}} \quad \text{(L'H)}$$

$$= \lim_{b\to 0^+} -b$$

$$= 0$$

3b) $\int_1^\infty \frac{dx}{x^3+1}$ $\quad \lim_{x\to\infty} \frac{\frac{1}{x^3}}{\frac{1}{x^3+1}} = \lim_{x\to\infty} \frac{x^3+1}{x^3} = \lim_{x\to\infty} 1+\frac{1}{x^3} = 1 < \infty,$

so $\int_1^\infty \frac{dx}{x^3+1} dx$ converges by the Limit Comparison Test, since $\int_1^\infty \frac{dx}{x^3}$ does.

c) $\int_\pi^\infty \frac{2+\cos x}{x} dx$ $\quad -1 \le \cos x \le 1 \Rightarrow 1 \le 2+\cos x \le 3$

Since $\int_\pi^\infty \frac{1}{x} dx$ diverges, and $\frac{2+\cos x}{x} \ge \frac{1}{x}$, we know that $\int_\pi^\infty \frac{2+\cos x}{x} dx$ also diverges by the Direct Comparison Test.

d) $\int_1^\infty \frac{\sqrt{x+1}}{x^2} dx$ $\quad \lim_{x\to\infty} \frac{\frac{\sqrt{x+1}}{x^2}}{\frac{1}{x^{3/2}}} = \lim_{x\to\infty} \frac{\sqrt{x^4+x^3}}{x^2}$

$= \lim_{x\to\infty} \frac{\sqrt{\frac{x^4}{x^4}+\frac{x^3}{x^4}}}{\frac{x^2}{x^2}} = 1 < \infty$

Since $\int_1^\infty \frac{1}{x^{3/2}} dx$ converges $(p = \frac{3}{2} > 1)$, we know that $\int_1^\infty \frac{\sqrt{x+1}}{x^2} dx$ also converges by the Limit Comparison Test.

e) $\int_1^\infty \frac{e^x}{x} dx$ diverges, since $\lim_{x\to\infty} \frac{e^x}{x} = \lim_{x\to\infty} \frac{e^x}{(1)}$ (L'H)

$= \infty > 0$

Since $\lim_{x\to\infty} \frac{e^x}{x} \ne 0$, $\int_1^\infty \frac{e^x}{x} dx$ must diverge.

f) $\int_1^\infty \frac{x}{e^x} dx = \lim_{b\to\infty} \int_1^b x e^{-x} dx$ $\quad u = x \quad v = -e^{-x}$; $du = dx \quad dv = e^{-x} dx$

$= \lim_{b\to\infty} -xe^{-x}\Big|_1^b + \int_1^b +e^{-x} dx$

$= \lim_{b\to\infty} -be^{-b} + e^{-1} - e^{-x}\Big|_1^b$

$= \lim_{b\to\infty} -be^{-b} + e^{-1} - e^{-b} + e^{-1}$

$= 0 + 2e^{-1} - 0 = 2e^{-1}$ (Converges)

4) $\int_2^4 \frac{1}{x-1}\,dx$, $n = 8$

$f(x) = \frac{1}{x-1} = (x-1)^{-1}$

$f'(x) = -(x-1)^{-2}$

$f''(x) = 2(x-1)^{-3}$

$f'''(x) = -6(x-1)^{-4}$

$f^{(4)}(x) = 24(x-1)^{-5}$

$= \frac{24}{(x-1)^5} \leq \frac{24}{(2-1)^5} = 24 \;(M)$ on $[2,4]$

Hence, $|E_s| \leq \frac{24(4-2)^5}{180(8)^4} = \frac{768}{737{,}280} = \frac{1}{960}$ $(\approx .00104)$

5) c

6) a

7) d

8) a

9) a