Math 224.02 Name:_____
Lab 6 - Sections 8.8, 6.1, & 6.2
October 8, 2009

1. Evaluate the following integrals without using tables.

   (a) $\displaystyle\int_0^1 (-\ln x)\, dx$

   (b) $\displaystyle\int_0^1 \frac{4r\, dr}{\sqrt{1-r^4}}$

2. Use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence.

   (a) $\displaystyle\int_{-1}^1 \ln|x|\, dx$

   (b) $\displaystyle\int_1^\infty \frac{dx}{x^3+1}$

   (c) $\displaystyle\int_\pi^\infty \frac{2+\cos x}{x}\, dx$

   (d) $\displaystyle\int_1^\infty \frac{\sqrt{x+1}}{x^2}\, dx$

   (e) $\displaystyle\int_1^\infty \frac{e^x}{x}\, dx$

   (f) $\displaystyle\int_1^\infty \frac{x}{e^x}\, dx$

3. (a) Sketch the region bounded by the graph of $y = x^3$ and $y = x$. Set up the integral, and calculate the area of the region.

   (b) Consider the portion of the region described in part (a) that lies in the first quadrant. Determine the volume obtained by rotating this region about the x-axis.

4. (a) Complete the following:
If S is the solid obtained by revolving the plane region R bounded by $y = f(x), \quad y = 0, \quad x = a$, and $x = b$ about the line $y = k$, sketch a graph of such a solid and a typical cross-section. The cross-sectional area is given by $A(x) =$ _____. The basic volume formula for this volume of revolution is

$$V = \int_{\_\_}^{\overline{\quad}} \_\_\_\_\_ \, dx.$$

(b) Complete the following:
If S is the solid obtained by revolving the plane region R bounded by $x = g(y), \quad x = 0, \quad y = c$, and $y = d$ about the line $x = h$, sketch a graph of such a solid and a typical cross-section. The cross-sectional area is given by $A(y) =$ _____. The basic volume formula for this volume of revolution is

$$V = \int_{\_\_}^{\overline{\quad}} \_\_\_\_\_ \, dy.$$

5. Sketch the region bounded by $y = \sqrt{x}$, the x-axis, and the line $x = 1$.

(a) i. Sketch the region obtained by rotating this region about the x-axis.
ii. Sketch a typical cross-section.
iii. Find an equation, $A(x)$, that describes the area of this cross-section.
iv. Find the limits of integration.
v. Set up the integral and evaluate it.

(b) i. Sketch the region obtained by rotating this region about the y-axis.
ii. Set up an integral using the washer method and evaluate it.
iii. Set up an integral using cylindrical shells and evaluate it.

6. Each integral represents the volume of a solid using disks/washers. Describe the solid.

(a) $\pi \int_0^{\pi/4} \tan^2 x \, dx$

(b) $\pi \int_1^2 y^6 \, dy$

(c) $\pi \int_0^1 (x - x^2) \, dx$