Math 224.02 Name:______________________________

Lab 7 - Sections 6.2, 6.3 & 6.4

October 15, 2009

1. (a) Sketch the region bounded by the graph of $y = x^3$ and $y = x$. Set up the integral, and calculate the area of the region.

   (b) Consider the portion of the region described in part (a) that lies in the first quadrant. Determine the volume obtained by rotating this region about the x-axis.

2. (a) Complete the following:
   If S is the solid obtained by revolving the plane region R bounded by $y = f(x)$, $y = 0$, $x = a$, and $x = b$ about the line $y = k$, sketch a graph of such a solid and a typical cross-section. The cross-sectional area is given by $A(x) =$ __________. The basic volume formula for this volume of revolution is

$$V = \int_{\_\_}^{\_\_} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \, dx.$$

   (b) Complete the following:
   If S is the solid obtained by revolving the plane region R bounded by $x = g(y)$, $x = 0$, $y = c$, and $y = d$ about the line $x = h$, sketch a graph of such a solid and a typical cross-section. The cross-sectional area is given by $A(y) =$ __________. The basic volume formula for this volume of revolution is

$$V = \int_{\_\_}^{\_\_} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \, dy.$$

3. Sketch the region bounded by $y = \sqrt{x}$, the x-axis, and the line $x = 1$.

   (a) i. Sketch the region obtained by rotating this region about the x-axis.
       ii. Sketch a typical cross-section.
       iii. Find an equation, $A(x)$, that describes the area of this cross-section.
       iv. Find the limits of integration.
       v. Set up the integral and evaluate it.

   (b) i. Sketch the region obtained by rotating this region about the y-axis.
       ii. Set up an integral using the washer method and evaluate it.
       iii. Set up an integral using cylindrical shells and evaluate it.

4. Find the lengths of the following curves.

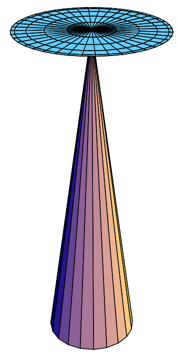
   (a) $y = x^{1/2} - (1/3)x^{3/2}, \quad 1 \leq x \leq 4$

   (b) $y = x^2 - \dfrac{\ln x}{8}, \quad 1 \leq x \leq 2$

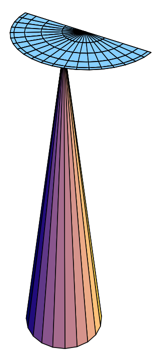
   (c) $x = 5\cos t - \cos 5t, \quad y = 5\sin t - \sin 5t, \quad 0 \leq t \leq \pi/2$

5. Two children weighing 50 pounds and 75 pounds are going to play on a seesaw that is 10 feet long. Where should the fulcrum be placed so that the seesaw will balance.

6. (a) An art student wishes to create a unique and interesting sculpture (pictured below). The student takes a circular metal disk two feet in diameter and will drill a small hole so that when the disk is placed atop a spike stuck in the hole, the disk will balance. Where on the disk should the student drill the hole so that it will balance (This one's a gimme.)



(b) It occurs to the student that the sculpture would look much cooler with half the disk atop the spike (pictured below). Not knowing about integrals, the artist drills a hole midway between the center and the edge, puts it on the spike, and it falls off! Where should the artist have drilled the hole?



(c) Now that the artist has made the mistake in part (b), she decides that rather than drill the second hole, she will cut the piece with the hole in it in such a way that it will balance on the spike at the point of the hole. Explain to the artist how the disk should be cut.