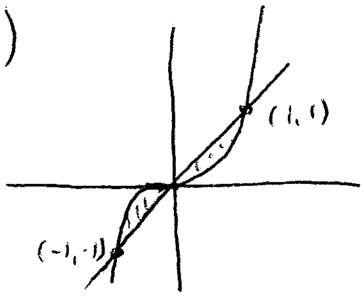
1a)



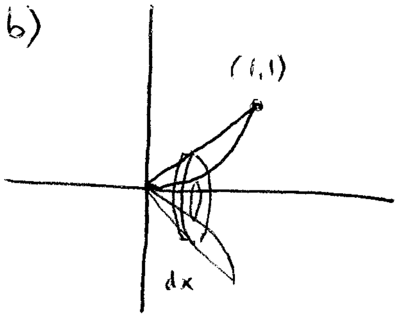
$$\text{Area} = \int_{-1}^{0} x^3 - x\,dx + \int_0^1 x - x^3\,dx$$

$$= 2\int_0^1 x - x^3\,dx$$

$$= 2\left(\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right|_0^1$$

$$= 2\left(\tfrac{1}{2} - \tfrac{1}{4}\right) = 2\left(\tfrac{1}{4}\right) = \tfrac{1}{2}$$

b)



$$\text{Volume} = \int_0^1 \pi\left(x^2 - (x^3)^2\right)dx$$

$$= \pi\int_0^1 x^2 - x^6\,dx$$

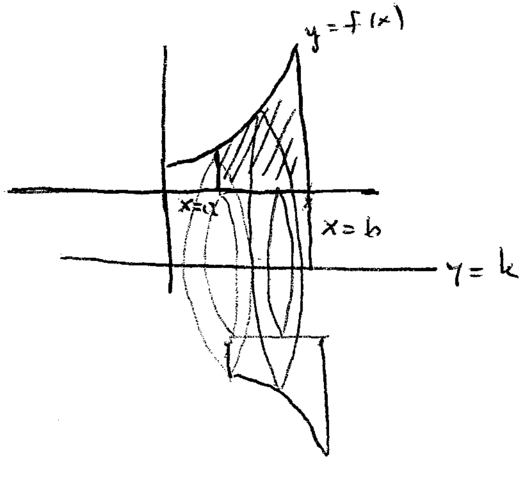
$$= \pi\left(\tfrac{1}{3}x^3 - \tfrac{1}{7}x^7\right|_0^1$$

$$= \pi\left(\tfrac{1}{3} - \tfrac{1}{7}\right)$$

$$= \tfrac{4}{21}\pi$$

2 a) $A(x) = \pi\left[(f(x)-k)^2 - (0-k)^2\right]$

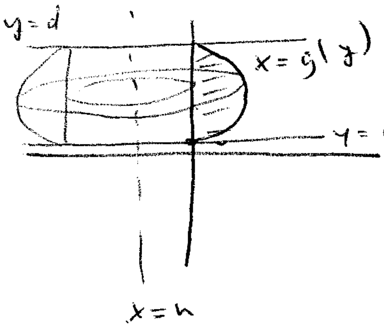
$= \pi\left[(f(x)-k)^2 - k^2\right]$



$$V = \int_a^b \pi\left[(f(x)-k)^2 - k^2\right]dx$$

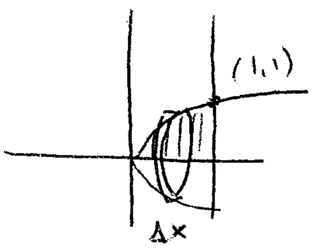
b) $A(y) = \pi\left[(g(y)-h)^2 - (0-h)^2\right]$

$= \pi\left[(g(y)-h)^2 - h^2\right]$



$$V = \int_c^d \pi\left[(g(y)-h)^2 - h^2\right]dy$$

3 a)



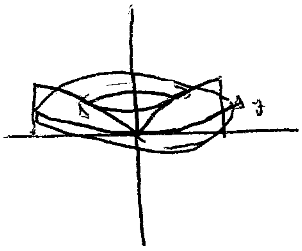
$A(x) = \pi(\sqrt{x})^2$

$$V = \int_0^1 \pi(\sqrt{x})^2\,dx = \pi\int_0^1 x\,dx$$

$= \frac{1}{2}\pi x^2\Big|_0^1$

$= \frac{1}{2}\pi$

b)

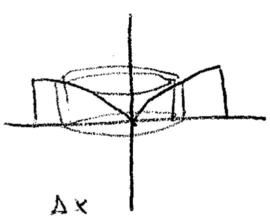


$y = \sqrt{x} \Rightarrow x = y^2$

Washers:

$$V = \int_0^1 \pi\left(1^2 - (y^2)^2\right)dy$$

$= \pi\int_0^1 1 - y^4\,dy = \pi\left(y - \frac{1}{5}y^5\Big|_0^1 = \frac{4}{5}\pi\right.$



Shells

$$V = 2\pi\int_0^1 x\cdot\sqrt{x}\,dx$$

$= 2\pi\int_0^1 x^{3/2}\,dx$

$= 2\pi\cdot\frac{2}{5}x^{5/2}\Big|_0^1$

$= \frac{4}{5}\pi$

4a) $y = x^{1/2} - \frac{1}{3}x^{3/2}$ $\qquad 1 \le x \le 4$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}$

$L = \int_1^4 \sqrt{1 + \left(\frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2}\right)^2}\, dx$

$= \int_1^4 \sqrt{1 + \frac{1}{4x} - 2\left(\frac{1}{4}\right) + \frac{x}{4}}\, dx$

$= \int_1^4 \sqrt{\frac{1}{4x} + \frac{1}{2} + \frac{x}{4}}\, dx \quad = \int_1^4 \sqrt{\left(\frac{1}{2\sqrt{x}} + \frac{\sqrt{x}}{2}\right)^2}\, dx$

$= \int_1^4 \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2}\, dx$

$= x^{1/2} + \frac{1}{3}x^{3/2} \Big|_1^4$

$= 2 + \frac{1}{3}(8) - 1 - \frac{1}{3} \quad = 1 + \frac{7}{3}$

b) $y = x^2 - \frac{\ln x}{8}$ $\qquad 1 \le x \le 2$

$\frac{dy}{dx} = 2x - \frac{1}{8x} \quad \Rightarrow 1 + \left(2x - \frac{1}{8x}\right)^2 = 1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$

$= 4x^2 + 1 + \frac{1}{64x^2}$

$= \left(2x + \frac{1}{8x}\right)^2$

$L = \int_1^2 \sqrt{\left(2x + \frac{1}{8x}\right)^2}\, dx \quad = \int_1^2 2x + \frac{1}{8}x^{-1}\, dx$

$= x^2 + \frac{1}{8}\ln x \Big|_1^2$

$= 4 + \frac{1}{8}\ln 2 - 1 - 0$

$= 3 + \frac{1}{8}\ln 2$

c) $x = 5\cos t - \cos 5t \quad , \quad y = 5\sin t - \sin 5t \qquad 0 \le t \le \frac{\pi}{2}$

$\frac{dx}{dt} = -5\sin t + 5\sin 5t \qquad \frac{dy}{dt} = 5\cos t - 5\cos 5t$

$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (-5\sin t + 5\sin 5t)^2 + (5\cos t - 5\cos 5t)^2$

$= 25\sin^2 t - 50\sin t \sin 5t + 25\sin^2 5t + 25\cos^2 t - 50\cos t \cos 5t$
$+ 25\cos^2 5t$

$= 25(\sin^2 t + \cos^2 t) + 25(\sin^2 5t + \cos^2 5t) - 50(\sin t \sin 5t + \cos t \cos 5t)$

$= 25(1) + 25(1) - 50\cos 4t$

$= 50(1 - \cos 4t)$

$\left(\begin{aligned} \cos 4t &= \cos(5t - t) \\ &= \cos 5t \cos t + \sin 5t \sin t \end{aligned}\right)$

Trig identity

$L = \int_0^{\pi/2} \sqrt{50 \cdot 2\sin^2 2t}\, dt$

$\left(\begin{aligned} 1 - \cos 4t = 2\sin^2 2t \end{aligned}\right)$

Half-angle formula

$= 10 \int_0^{\pi/2} \sin 2t\, dt$

$= -5\cos 2t \Big|_0^{\pi/2}$

$= -5(-1 - 1)$

$= 10$

5) 

$50 \cdot x = 75 \cdot (10 - x)$

$50x = 750 - 75x$

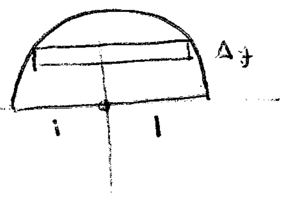
$125x = 750$

$x = \frac{750}{125} = 6$

The fulcrum should be placed 6 ft from the 50 lb child (and 4 ft from the 75 lb child)

6a) The hole should be placed in the center of the disk.

b)



$x^2+y^2=1$

$x=\sqrt{1-y^2}$

length $= 2x = 2\sqrt{1-y^2}$

area: $dA = 2\sqrt{1-y^2}\,dy$

Mass: $M = \int_0^1 \delta \cdot 2\sqrt{1-y^2}\,dy$

$= 2\delta \int_0^1 \sqrt{1-y^2}\,dy$ (Area of a quarter circle)

$= 2\delta \left(\frac{1}{2}\pi\right) = \delta\pi$

area of semi-unit-circle

$M_x = \int_0^1 \delta y \cdot 2\sqrt{1-y^2}\,dy$

$u = 1-y^2$

$-du = 2y\,dy$

$= \delta \int_1^0 u^{1/2} \cdot -du$

$= \delta \int_0^1 u^{1/2}\,du$

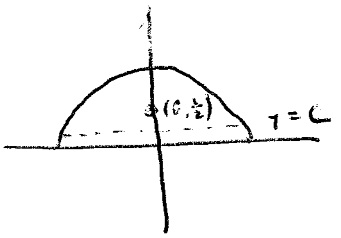
$= \delta \frac{2}{3} u^{3/2} \Big|_0^1$

$= \frac{2}{3}\delta$

$\bar{y} = \frac{M_x}{M} = \frac{\frac{2}{3}\delta}{\delta\pi} = \frac{2}{3\pi}$

The hole should be drilled at the center of mass: $(\bar{x},\bar{y}) = (0, \frac{2}{3\pi})$

c)



Here is one way to do it.

We need $\bar{y} = \frac{M_x}{M} = \frac{\delta\int_c^1 2y\sqrt{1-y^2}\,dy}{2\delta\int_c^1 \sqrt{1-y^2}\,dy}$

(Formula #24, $a=1$)

$= \frac{\frac{2}{3}(1-y^2)^{3/2}\Big|_c^1}{\frac{y}{2}\sqrt{1-y^2}+\frac{1}{2}\sin^{-1}y\Big|_c^1}$

This is a difficult equation to solve for c

$= \frac{-\frac{2}{3}(1-c^2)^{3/2}}{\frac{1}{2}\cdot\frac{\pi}{2} - \frac{c}{2}\sqrt{1-c^2} - \frac{1}{2}\sin^{-1}c}$

$= \frac{1}{2}$

Alternatively, you could cut it like this: