

1. Find the series' radius and interval of convergence.

(a) $\sum_{n=0}^{\infty} (x + 5)^n$

(b) $\sum_{n=1}^{\infty} \frac{(x - 1)^n}{\sqrt{n}}$

(c) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$

(d) $\sum_{n=1}^{\infty} \frac{(2x + 3)^{2n+1}}{n!}$

(e) $\sum_{n=1}^{\infty} (\ln n)x^n$

(f) $\sum_{n=0}^{\infty} n!(x - 4)^n$

2. Verify that the following sums are true. (Using Taylor series at the prescribed values)

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = \ln 2$

(b) $\sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{(2n + 1)!} \right] = \sin 1$

3. We know that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Use this to find the Taylor series for the following functions.

(a) $f(x) = e^{-x}$

(b) $g(x) = e^{3x}$

(c) $h(x) = xe^x$

(d) $f(x) = e^{2x} + e^{-2x}$

4. Determine the Taylor series for e^{2x} about $x = 0$ using each of methods:

(a) Use the definition of Taylor series, name that the n^{th} coefficient is $a_n = \frac{f^{(n)}(0)}{n!}$.

(b) Replace x by $2x$ in the Taylor series for e^x .

(c) Multiply the Taylor series for e^x by itself, as $e^{2x} = e^x \cdot e^x$.