Section 6.2 - Volumes by Cylindrical Shells

1. Shell Formula for Revolution about a Vertical Line:
   The volume of the solid formed by rotating the region between the x-axis and the graph of a continuous function $y = f(x) \geq 0$, $L \leq a \leq x \leq b$, about a vertical line $x = L$ is

   $$V = \int_a^b 2\pi(\text{shell radius})(\text{shell height})\, dx.$$

2. Example #1: (Thomas, Section 6.2, pr. 2)
   Use the shell method to find the volume of the solid obtained by rotating the region bounded by $y = 2 - \dfrac{x^2}{4}$ and the y-axis, $0 \leq x \leq 2$ about the y-axis. (Ans. 6π)

3. Example #2: (Thomas, Section 6.2, pr. 4)
   Use the shell method to find the volume of the solid obtained by rotating the region bounded by $x = 3 - y^2$ and the lines $y = \sqrt{3}$ and $x = 3$ $(0 \leq y \leq \sqrt{3})$ about the x-axis. (Ans. $9\pi/2$)

4. Example #3: (Thomas, Section 6.2, pr. 20)
   Use the shell method to find the volume of the solid generated by rotating the region bounded by $y = x$, $y = 2x$, and $y = 2$ about the x-axis. (Ans. $2\pi \int_0^2 y(y - \frac{1}{2}y)\, dy = 8\pi/3$)

5. Example #4: (Thomas, Section 6.2, pr. 24)
   Use the shell method to find the volume of the solid obtained by rotating the region above the x-axis that is bounded by $x = \dfrac{y^4}{4} - \dfrac{y^2}{2}$ and $x = \dfrac{y^2}{2}$ about
   (a) The x-axis. (Ans. $2\pi \int_0^2 y(y^2 - \frac{y^4}{4})\, dy = 8\pi/3$)
   (b) The line $y = 2$. (Ans. $2\pi \int_0^2 (2 - y)(y^2 - \frac{y^4}{4})\, dy = 8\pi/5$)
   (c) The line $y = 5$. (Ans. $2\pi \int_0^2 (5 - y)(y^2 - \frac{y^4}{4})\, dy = 8\pi$)

6. Example #5: (Thomas, Section 6.2, pr. 28)
   Consider the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$. Determine whether it would be better to use washers or shells to determine the volume of the solid obtained by rotating the region about the
   (a) The x-axis.
   (b) The y-axis.
   (c) The line $x = 4$.
   (d) The line $y = 2$.