Section 8.2 - Integration by Parts

1. Note: Integral of a product is **NOT** the product of the integrals!
2. Imagine that we want to solve the following integral:

$$\int x \sin x \; dx.$$

Observe that

$$\frac{d}{dx}(x \cos x) = \cos x - x \sin x.$$

It follows that

$$\int \cos x - x \sin x \; dx = x \cos x + C, \text{ or}$$

$$\int x \sin x \; dx = -x \cos x + \int \cos x \; dx, \text{ or}$$

$$\int x \sin x \; dx = -x \cos x + \sin x \; dx.$$

3. In general,

$$\frac{d}{dx}(u(x) \cdot v(x)) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

$$\Rightarrow \int u'(x) \cdot v(x) + u(x) \cdot v'(x) \; dx = u(x) \cdot v(x)$$

$$\Rightarrow \int u \cdot v' \; dx = u \cdot v - \int u' \cdot v \; dx.$$

4. Pick u first. Order of precedence:
   (a) Something involving ln
   (b) Powers of your variable (x^n)

   Some things simplify when you differentiate, others when you integrate.

5. Examples:
   (a) $\int xe^x \; dx$
   (b) $\int \ln x \; dx$
   (c) $\int_0^1 x^2 \cdot e^x \; dx$ (Not changing variable, hence don't change limits.)
   (d) $\int e^x \cdot \sin x \; dx$

(e) Prove the reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n} \cos x \cdot \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \, dx,$$

where $n \geq 2$ is an integer.

6. Integration by Parts Formula for Definite Integrals:

$$\int_a^b f(x)g'(x) \, dx = f(x)g(x)|_a^b - \int_a^b f'(x)g(x) \, dx$$

7. When to use Integration by Parts

- Integral of a product (that can't be removed by substitution).
- You have something you know how to differentiate but not how to integrate ($\ln x$)

8. Example: Tabular Integration

$$\int x^5 e^x \, dx$$