Section 8.5 - Trigonometric Substitutions

1. We want to integrate $\int \frac{1}{\sqrt{9-x^2}}\,dx$.
    - No other integration techniques that we've looked at to this point apply.
    - We observe that it looks similar to $\arcsin x$. $\left(\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C\right)$
    - Also observe the Pythagorean Theorem (Sum/Difference of 2 squares under a radical).
    - Review right-triangle trigonometry.
    - Solve the integral.

2. Example #2:
$$\int \frac{dx}{x^2\sqrt{4-x^2}}.$$
Hint: Use $\tan^2\theta = \sec^2\theta - 1$.

3. Example #3:
$$\int \frac{dx}{x^2\sqrt{x^2+25}}.$$

4. Example #4:
$$\int \frac{x}{x^2-4x+8}\,dx.$$
Hint: Complete the square, u-substitution, then split up integrals.

5. Example #5: Determine the area of the ellipse $x^2+9y^2=16$
Half-angle formulas:
$$\sin^2 x = \frac{1-\cos 2x}{2}$$
$$\cos^2 x = \frac{1+\cos 2x}{2}.$$

6. Example #6: Solve the initial value problem
$$\sqrt{x^2-9}\,\frac{dy}{dx} = 1, \qquad x > 3, \qquad y(5) = \ln 3$$