Section 8.8 - Comparison of Improper Integrals

1. Sometimes it's tough to determine the exact value of an improper integral, but it may be possible to compare a given integral to one who's behavior we already know.

2. Example: Determine whether $\displaystyle\int_1^\infty \frac{1}{\sqrt[3]{x^4+3}}\,dx$ converges.

3. The Comparison Test for $\displaystyle\int_a^\infty f(x)\,dx$:

   - If $0 \le f(x) \le g(x)$ and $\displaystyle\int_a^\infty g(x)\,dx$ converges, then $\displaystyle\int_a^\infty f(x)\,dx$ converges.
   - If $0 \le g(x) \le f(x)$ and $\displaystyle\int_a^\infty g(x)\,dx$ diverges, then $\displaystyle\int_a^\infty f(x)\,dx$ diverges.

4. Example: Determine whether $\displaystyle\int_0^\infty \frac{x^3+3x^2+4}{x^4+2x+11}\,dx$ converges.

5. Useful Integrals for Comparison:

   - $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converges for $p > 1$ and diverges for $p \le 1$.
   - $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ converges for $p < 1$ and diverges for $p \ge 1$.
   - $\displaystyle\int_0^\infty e^{-ax}\,dx$ converges for $a > 0$.

6. Decide if the improper integral converges or diverges.

   (a) $\displaystyle\int_0^1 \frac{d\theta}{\sqrt{\theta^3+\theta}}$

   (b) $\displaystyle\int_1^\infty \frac{\cos\phi}{\phi^2}\,d\phi$

   (c) $\displaystyle\int_0^\infty \frac{dz}{e^z+2^z}$