

Exam 2

Name: \_\_\_\_\_

Math 224.01  
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The point values for each problem are given below.

YOU MUST SHOW YOUR WORK/JUSTIFY YOUR ANSWER TO RECEIVE FULL CREDIT FOR A PROBLEM.

| <b>Question</b> | <b>Points Earned</b> | <b>Points Possible</b> |
|-----------------|----------------------|------------------------|
| 1               |                      | 10                     |
| 2               |                      | 5                      |
| 3               |                      | 5                      |
| 4               |                      | 7                      |
| 5               |                      | 15                     |
| 6               |                      | 10                     |
| 7               |                      | 5                      |
| 8               |                      | 10                     |
| 9               |                      | 10                     |
| 10              |                      | 10                     |
| Total           |                      | 85                     |

1. Estimate the integral  $\int_0^2 x^4 + 6 \, dx$  using the following methods.

(a) Trapezoid Rule with  $n = 4$  steps.

(b) Simpson's Rule with  $n = 4$  steps.

2. Estimate the minimum number of subintervals needed to approximate the integral

$$\int_3^4 (2x^3 + 5x) \, dx$$

with an error of magnitude less than  $10^{-4}$  using the Trapezoidal Rule.

Note: Recall that

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}.$$

3. Evaluate the integral  $\int \frac{\sqrt{6x - x^2}}{x} dx$  using a table of integrals.

4. Evaluate the integral  $\int \frac{dx}{x \ln x \sqrt{\ln x + 3}}$  by making a substitution and then using a table of integrals.

5. Evaluate the following integrals or state that they diverge.

(a)  $\int_1^{\infty} e^{-3x} dx$

(b)  $\int_1^{\infty} \frac{\ln x}{x} dx$

(c)  $\int_0^1 \frac{x^4 + 1}{x} dx$

6. Recall that

- $\int_1^{\infty} \frac{1}{x^p} dx$  converges for  $p > 1$  and diverges for  $p \leq 1$ .
- $\int_0^1 \frac{1}{x^p} dx$  converges for  $p < 1$  and diverges for  $p \geq 1$ .
- $\int_0^{\infty} e^{-ax} dx$  converges for  $a > 0$ .

Determine whether the following integrals converge or diverge using integration, the Direct Comparison Test, or the Limit Comparison Test. You do **not** need to evaluate the integrals.

(a)  $\int_1^{\infty} \frac{1}{e^{3t} + t + 1} dt.$

(b)  $\int_1^{\infty} e^{-x} \sin x dx.$

7. Set up but **do not evaluate** the integral that determines the arc length of the function  $f(x) = \sqrt{16 - x^2}$  from  $x = 0$  to  $x = 4$ .

8. Consider the region bounded by  $y = e^x$ , the  $x$ -axis, and the lines  $x = 0$  and  $x = 3$ . Find the volume of the solid obtained by rotating the region about the  $x$ -axis.
9. The region enclosed by the curves  $y = x^2$  and  $y = 2x$  is rotated about the  $y$ -axis. Find the volume of the solid generated by this rotation.
10. Use the shell method to write a definite integral representing the volume of the solid obtained by revolving the region bounded by  $x = 3y - y^2$  and the  $y$ -axis about the line  $y = 4$ . (You do **not** need to evaluate the integral.)