Exam 3 Name:_____

Math 224.01
November 10, 2008

The point values for each problem are given below.
YOU MUST SHOW YOUR WORK/JUSTIFY YOUR ANSWER TO RECEIVE FULL CREDIT FOR A PROBLEM.

Question	**Points Earned**	**Points Possible**
1		10
2		10
3		10
4		16
5		15
6		8
		8
		8
		8
		8
Total		100

Common Limits:

1. $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\lim_{n\to\infty} x^{1/n} = 1$ (any fixed x)
4. $\lim_{n\to\infty} x^n = 0$ ($|x| < 1$)
5. $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$ (any fixed x)
6. $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ (any fixed x)

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1. A thin plate with constant density 3 g/cm^2, has a shape bounded by the curve $y = \sqrt{x}$ and the x-axis with $0 \leq x \leq 1$ and x, y in cm.

   (a) Find the total mass of the plate.

   (b) Find the center of mass of the plate.

2. A cable that weighs 3 lb/ft is used to lift 600 lbs of coal up a mineshaft 500 ft deep. Determine the work necessary to lift the cable and coal completely out of the mineshaft.

3. A fuel oil tank is an upright cylinder, buried so that its circular top is 8 feet beneath ground level. The tank has a radius of 5 feet and is 12 feet high, and the tank is full of oil. Calculate the work required to pump all of the oil to the surface, assuming that the oil weighs 50 lb/ft^3.

4. Find the sum of the following series.

   (a) $\displaystyle\sum_{i=3}^{\infty} \left(\frac{3}{4}\right)^i$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{9}{(3n-1)(3n+2)}$.

5. Determine whether the sequences whose nth terms appear below converge or diverge. Find the limit of each convergent sequence.

(a) $a_n = \dfrac{\ln(n^2)}{n}$

(b) $a_n = \sqrt[n]{\dfrac{3^n}{n}}$

(c) $a_n = \dfrac{(n+1)!}{n!}$

6. You are to complete only **5** of the following 6 parts. If you do not clearly mark which 5 you wish to have graded (or which 1 you do not), I will grade the first 5 parts. In each case, determine whether the specified series converges or diverges. Be sure to justify your answers.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{4n+4}{n^2+2n+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \frac{1}{n^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 2}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$