Math 224.02 Name: Answer Key

Quiz 11 - December 3, 2009

1. Consider the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+3}}.$$

(a) Find the series' radius and interval of convergence.

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} x^{n+1}}{\sqrt{(n+1)^2+3}}}{\frac{(-1)^n x^n}{\sqrt{n^2+3}}} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{\sqrt{(n+1)^2+3}} \cdot \frac{\sqrt{n^2+3}}{x^n} \right|$$

$$= \lim_{n\to\infty} |x| \cdot \frac{\sqrt{n^2+3}}{\sqrt{(n+1)^2+3}}$$

$$= |x| \cdot 1$$

(2) $= |x| < \boxed{1 = R}$ (1)

$\Rightarrow$ Converges absolutely on $(-1,1)$ by the Ratio Test.

$x = -1$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{\sqrt{n^2+3}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+3}}$$

diverges by comparison to $\sum_{n=0}^{\infty} \frac{1}{n}$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$$

converges by AST

(1) since $\lim_{n\to\infty} \frac{1}{\sqrt{n^2+3}} = 0$

$\boxed{\text{Interval: } (-1, 1]}$

(b) For what values of x does the series converge absolutely?

(1/2) $(-1,1)$

(c) For what values of x does the series converge conditionally?

(1/2) at $x = 1$

2. Find the radius of convergence for the series

$$\sum_{n=1}^{\infty} n^n x^n$$

(2) $$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1} x^{n+1}}{n^n x^n} \right| = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{n^n} \cdot |x| = \infty > 1$$

$\Rightarrow R = 0$

(converges only when $x = 0$)

3. Find the Maclaurin series (Taylor series at $x = 0$) for the function

$$e^{x/2}.$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\Rightarrow e^{x/2} = \sum_{n=0}^{\infty} \frac{(\frac{x}{2})^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{2^n \cdot n!}$$

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or

$f(x) = e^{x/2} \Rightarrow f(0) = 1$

$f'(x) = \frac{1}{2}e^{x/2} \Rightarrow f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{4}e^{x/2} \Rightarrow f''(0) = \frac{1}{4}$

$f'''(x) = \frac{1}{8}e^{x/2} \Rightarrow f'''(0) = \frac{1}{8}$

$f^{(4)}(x) = \frac{1}{16}e^{x/2} \Rightarrow f^{(4)}(0) = \frac{1}{16}$

$\vdots$

$f^{(n)}(x) = \frac{1}{2^n}e^{x/2} \Rightarrow f^{(n)}(0) = \frac{1}{2^n}$

$$e^{x/2} = 1 + \frac{1}{2}x + \frac{\frac{1}{4}}{2!}x^2 + \frac{\frac{1}{8}}{3!}x^3 + \frac{\frac{1}{16}}{4!}x^4 + \cdots + \frac{\frac{1}{2^n}}{n!}x^n + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{1}{2^n n!} x^n$$