

Exam 1

Name: _____

Math 301.01
March 1, 2007

Question	Points Earned	Points Possible
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
Bonus		3
Total		80

1. Use truth tables to determine whether the following statement forms are logically equivalent.

$$\sim (p \rightarrow q) \quad \text{and} \quad p \wedge \sim q.$$

2. Consider the following statement:

$$\forall n \in \mathbb{Z}, \text{ if } 9|n \text{ then } 9|n^2.$$

- (a) Determine the negation of the statement.
- (b) Determine the converse of the statement.
- (c) Determine the inverse of the statement.
- (d) Determine the contrapositive of the statement.
- (e) Which of the following statements are true for the example considered in this problem: the original statement, the negation, the converse, the inverse, and the contrapositive.

3. You are given the following rules of inference.

Modus Ponens	$p \rightarrow q$ p $\therefore q$	Elimination	a. $p \vee q$ $\sim q$ $\therefore p$	b. $p \vee q$ $\sim p$ $\therefore q$	
Modus Tollens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Transitivity	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$		
Generalization	a. p $\therefore p \vee q$	b. q $\therefore p \vee q$	Specialization	a. $p \wedge q$ $\therefore p$	b. $p \wedge q$ $\therefore q$
Proof by Division into Cases	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$	Conjunction	p q $\therefore p \wedge q$		

A set of premises and a conclusion are given. Use the rules of inference above to deduce the conclusion from the premises, giving a reason for each step.

- a. $p \wedge s \rightarrow \sim u$
- b. $q \wedge s$
- c. $w \rightarrow z$
- d. $\sim r$
- e. $\sim u \vee v \rightarrow w \vee y$
- f. $y \rightarrow z$
- g. $p \vee r$
- h. $\therefore z$

4. Prove the following statement directly from known definitions.

The difference of any even integer minus any odd integer is odd.

5. Disprove the following statement by giving a counterexample:

(a) For all integers m and n , if $4m + n$ is odd then m and n are both odd.

(b) For all integers a , b , and c , if a is a factor of c then ab is a factor of c .

6. Suppose that you have already proved the following properties of odd and even integers:

- (a) The sum, product, and difference of any two even integers are even.
- (b) The sum and difference of any two odd integers are even.
- (c) The product of any two odd integers is odd.
- (d) The product of any even integer and any odd integer is even.
- (e) The sum of any odd integer and any even integer is odd.
- (f) The difference of any odd integer minus any even integer is odd.
- (g) The difference of any even integer minus any odd integer is odd.

Use the properties above to prove that if a is any odd integer and b is any even integer then $\frac{a^2 + 3b - 1}{2}$ is an integer.

7. For all $a, b, c \in \mathbb{Z}$, prove that if $ab|c$ then $a|c$.

8. Prove that the square of any integer has the form $4k$ or $4k + 1$ for some integer k .

Bonus: Find the mistake in the following “proof” that $\lfloor n/2 \rfloor = \frac{n-1}{2}$ if n is an odd integer.

“**pf.** Suppose n is any odd integer. Then $n = 2k + 1$ for some integer k . Consequently,

$$\left\lfloor \frac{2k+1}{2} \right\rfloor = \frac{(2k+1) - 1}{2} = \frac{2k}{2} = k.$$

But $n = 2k + 1 \Rightarrow k = \frac{n-1}{2}$. Hence, by substitution, $\lfloor n/2 \rfloor = \frac{n-1}{2}$.”