# Section 1.2: Conditional Statements

1. Conditional Statement: 2 parts (hypothesis/antecedent and conclusion/consequent)

   - If $p$ then $q$.
   - $p \rightarrow q$.
   - $p \Rightarrow q$.
   - $p$ implies $q$.
   - $q$ can be logically deduced from $p$.
   - $p$ is sufficient for $q$.

   Order of operations: $\rightarrow$ is last.

2. **Vacuously true**: true because the hypotheses are false.
   If my hair is on fire, then class is canceled.

3. Example: Construct truth tables for $\sim p \vee q$ and $p \rightarrow q$.

4. Example: Construct a truth table to show $p \wedge q \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$.

5. Negation of a conditional statement: $\sim (p \rightarrow q) \equiv p \wedge \sim q$.
   Note: The negation of an if-then statement **does not start** with the word 'if.'

6. The **contrapositive** of $p \rightarrow q$ is $\sim q \rightarrow \sim p$. (Show equivalence)

7. Converse and Inverse:

   (a) The **converse** of $p \rightarrow q$ is $q \rightarrow p$. (Not equivalent)

   (b) The **inverse** of $p \rightarrow q$ is $\sim p \rightarrow \sim q$.

   Note: A conditional statement may be true but its inverse or converse may still be false.

8. $p$ **only if** $q$ means "if not $q$ then not $p$" or equivalently $p \rightarrow q$.
   Note: Be careful to distinguish between "$p$ only if $q$" and "$p$ if $q$."

9. **Biconditional**: "if and only if" or $\Leftrightarrow$ or $\leftrightarrow$ or **iff**.
   Go through truth table for biconditional.

10. **Necessary** and **sufficient** conditions: converting to "If-Then" form.

11. Remarks:

    - In logic, a hypothesis and conclusion need not be related.
      Example: If I am over six feet tall, then Marietta is in Ohio.
    - In informal language, simple conditionals are often used to mean biconditionals.
      Example: If you go to the game, then I'll go to the game.