Section 1.3: Valid and Invalid Arguments

1. Definitions:
   - **Argument:** a sequence of statements ending in a conclusion.
   - **Argument form:** a sequence of statement forms.
   - **Premises:** all statements/statement forms except for the last (**conclusion**)
   - **Valid** (argument form): no matter what the premises are, if they're true, then the conclusion must be true. Argument is valid if its form is valid. (Notation: $\therefore$)

2. Testing an Argument Form for Validity:
   (a) Identify the premises and conclusion.
   (b) Construct a truth table showing all truth values of premises and conclusion.
   (c) If there is a row where all premises are true and conclusion is false, then invalid.
   If every time the premises are true the conclusion is also true, then valid.

3. Example #1: Determine if the following argument form is valid or invalid. (p. 41, pr. 7)

$$\begin{array}{ll} & p \\ & p \rightarrow q \\ & \sim q \vee r \\ \therefore & r \end{array}$$

4. Definitions: (Give an example for each)
   (a) **Syllogism**: an argument form with two premises and a conclusion.
   (b) **Modus ponens** ("method of affirming"):

$$\begin{array}{ll} & p \rightarrow q \\ & p \\ \therefore & q \end{array}$$

   (c) **Modus tollens** ("method of denying"):

$$\begin{array}{ll} & p \rightarrow q \\ & \sim q \\ \therefore & \sim p \end{array}$$

   (d) **Rule of inference**: any valid argument form
   (e) Generalization:

$$\begin{array}{ll} & p \\ \therefore & p \vee q \end{array}$$

   (f) Specialization:

$$\begin{array}{ll} & p \wedge q \\ \therefore & p \end{array}$$

(g) Elimination:

$$\begin{array}{rl} & p \vee q \\ & \sim q \\ \therefore & p \end{array}$$

(h) Transitivity:

$$\begin{array}{rl} & p \to q \\ & q \to r \\ \therefore & p \to r \end{array}$$

(i) Division into Cases:

$$\begin{array}{rl} & p \vee q \\ & p \to r \\ & q \to r \\ \therefore & r \end{array}$$

5. Example #2: (Only if time permits-p. 42, pr. 37)
In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logic puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements and challenged the reader to figure out the location of the treasure.

   (a) If this house is next to the lake, then the treasure is not in the kitchen.
   (b) If the tree in the front yard is an elm, then the treasure is in the kitchen.
   (c) This house is next to a lake.
   (d) The tree in the front yard is an elm or the treasure is buried under the flagpole.
   (e) If the tree in the back yard is an oak, then the treasure is in the garage.

6. Example: A set of premises and a conclusion are given. Use valid argument forms to deduce the conclusion from the premises.

	#1		**#2**
(a)	$p \vee \sim s$	(a)	$r \vee s$
(b)	$p \to r \vee w$	(b)	$q \to \ \sim p$
(c)	$w \to v$	(c)	$w \vee s \to v$
(d)	$u \to (\sim v \vee \sim p)$	(d)	$r \to p$
(e)	s	(e)	$\sim q \vee r \to u \wedge w$
(f)	$r \to v$	(f)	$s \to p$
(g) $\therefore$	$\sim u$	(g) $\therefore$	v

7. **Fallacies**: errors in reasoning that result in an invalid argument.

   - Using ambiguous premises, and treating them as if they're unambiguous.
     Ex. Basketball players are tall.
   - Begging the question (assuming the conclusion is true without having derived it from the premises)
     Ex. Euthanasia.
   - Jumping to a conclusion (without adequate grounds).
     Ex. DNA found at crime scene.

   The last two are the two major errors committed by students first trying to prove.

- Converse error:

$$\begin{array}{rl} & p \to q \\ & q \\ \therefore & p \end{array}$$

- Inverse error:

$$\begin{array}{rl} & p \to q \\ & \sim p \\ \therefore & \sim q \end{array}$$

- Example where the argument is valid but the conclusion is false and vice versa.

8. Contradiction Rule:
   If assuming that p is false logically leads to a contraction, then p must be true.

$$\begin{array}{rl} & \sim p \to c \\ \therefore & p \end{array}$$