Section 2.2: Introduction to Predicates and Quantified Statements II

1. Negations of "for every" and "there exists" statements:
   - What is the negation of "All Marietta College professors are brilliant."
   - Note: Sometimes informal negations can be constructed by inserting not or do not into the original statement. Be careful. Consider "All Marietta College professors are not brilliant."
   - $\sim (\forall\ x \in D),\ Q(x) \equiv \exists\ x \in D$ s.t. $\sim Q(x)$.
   - What is the negation of "Some students like studying?"
   - $\sim (\exists\ x \in D$ s.t. $Q(x)) \equiv \forall\ x \in D, \sim Q(x)$.
2. Example #1: (p. 96, pr. 12) Is the proposed negation correct? If not, correct it.
   Statement: The product of any irrational number and any rational number is irrational.
   Negation: The product of any irrational number and any rational number is rational.
3. Negations of "if-then statement" is $\equiv$ to an *and* statement:
   - $\sim (\forall\ x, P(x) \to Q(x)) \equiv \exists\ x$ s.t. $P(x)$ and $\sim Q(x)$.
4. Example #2: (p. 96, 21, 23) Write a negation for each statement.
   (a) $\forall\ n \in \mathbb{Z}$, if n is a prime then n is odd or $n = 2$.
   (b) $\forall$ animals x, if x is a dog then x has paws and x has a tail.
5. Equivalent to De Morgan's laws:
   - Negation of *for all* is *there exists* and vice versa.
   - *for all* is a generalization of an *and* statement.
   - *there exists* is a generalization of an *or* statement.
6. Vacuous Truth of "for all" statements:
   - $\forall$ pink elephants ...
   - $\forall$ Cleveland Brown Super Bowl teams
7. Variants: Consider $\forall x \in D,\ P(x) \to Q(x)$.
   (a) **Contrapositive:** $\forall x \in D,\ \sim Q(x) \to \sim P(x)$.
   (b) **Converse:** $\forall x \in D,\ Q(x) \to P(x)$.
   (c) **Inverse:** $\forall x \in D,\ \sim P(x) \to \sim Q(x)$.
8. Example #3: (p. 96, pr. 32, 34) Find the contrapositive, converse, and inverse.
   (a) $\forall\ n \in \mathbb{Z}$, if n is a prime then n is odd or $n = 2$.
   (b) $\forall$ animals x, if x is a dog then x has paws and x has a tail.
9. Example #4: (p. 96, pr. 39) Rewrite in if-then form.
   Being divisible by 8 is a sufficient condition for being divisible by 4.
10. Example #5: (p. 97, pr. 45) Rewrite without using *necessary* or *sufficient.*
    Being a polynomial is not a sufficient condition for a function to have a real root.