

## Section 3.5: Direct Proof V: Floor and Ceiling

1.
  - $\lceil x \rceil =$  the unique integer  $n$  s.t.  $n - 1 < x \leq n$ .
  - $\lfloor x \rfloor =$  the unique integer  $n$  s.t.  $n \leq x < n + 1$ .

Go through positive and negative examples of each.

2. Example #1: Is the following true or false:  $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ ?

3. Theorem 3.5.1  $\forall x \in \mathbb{R}, m \in \mathbb{Z}, \lfloor x + m \rfloor = \lfloor x \rfloor + m$ .

Prove this.

4. Example #2: Prove or find a counterexample.

For all real numbers  $x$  and  $y$ ,  $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$

5. Example #3: Prove or find a counterexample.

For all odd integers  $n$   $\lceil \frac{n}{2} \rceil = \frac{n+1}{2}$ .